Persistence-based reconstruction

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A common problem in computational geometry is trying to reconstruct a structured object X (graph, submanifold, stratified space, etc) from a point set sampling this object. Common techniques for this require rather strong hypotheses on the object and the sampling (see for instance [2]). On the other hand, with much weaker hypotheses, persistent homology is able to extract information about X. It does not provide a reconstruction, but we can often reconstruct two objects $K \subset L$ such that the inclusion induces at the homology level a morphism whose rank matches the Betti numbers of X[4, 3]. It is tempting to try and find a reconstruction Y of X such that $K \subseteq Y \subseteq L$. However, such a reconstruction does not always exist, and determining whether it exists is already NP-hard[1]. This bad news does not mean that nothing can be done. The same paper shows that in some special cases (including α-complexes in 3D), the complexity drops to polynomial.

The goal of this internship is to explore further how a reconstruction can be extracted from a pair of nested simplicial complexes. There are several aspects that can be studied, depending on the tastes of the candidate and on which leads prove more fruitful.

- Devise an efficient algorithm to find Y such that $K \subseteq Y \subseteq L$ when it exists in the special case where it can be done in polynomial time. Devise heuristics for the NP-hard case, that can handle "easy" cases.
- Experimentally determine how often Y does not exist, in some specific contexts (α-complexes in 3D for instance). The goal is to determine if the non-existence of Y (and the NP-hardness) is the usual case, or if it is rare enough in realistic cases that it is worth trying to reconstruct this way.
- Devise examples where Y does not exist that look as realistic as possible, and satisfy stronger hypotheses. For instance, while we often consider an α-complex K and the corresponding 3α-complex L, considering a constant larger than 3 would be interesting.
- Try and find some stronger hypotheses (on the object, the sampling, how K and L are built, etc) that guarantee that Y exist. This is symmetric to the previous item, and the goal would be to narrow the gap between the two to understand precisely what causes issues.

- In cases where Y does not exist as a subcomplex, some surgery (splitting edges, thickening simplices, etc) might still allow to “fix” the problem and give some Z that is a topologically correct reconstruction of X. A few heuristics might let us decrease significantly the probability that this reconstruction technique fails. It may also be possible to prove that no pair $K \subset L$ resists the surgery techniques, with sufficient assumptions.

References

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