École Mathématique en Afrique

Topologie différentielle, géométrie algébrique et applications

28 mars - 9 avril, La Marsa, Tunisie

Persistance Topologique

théorie et applications

Steve Oudot

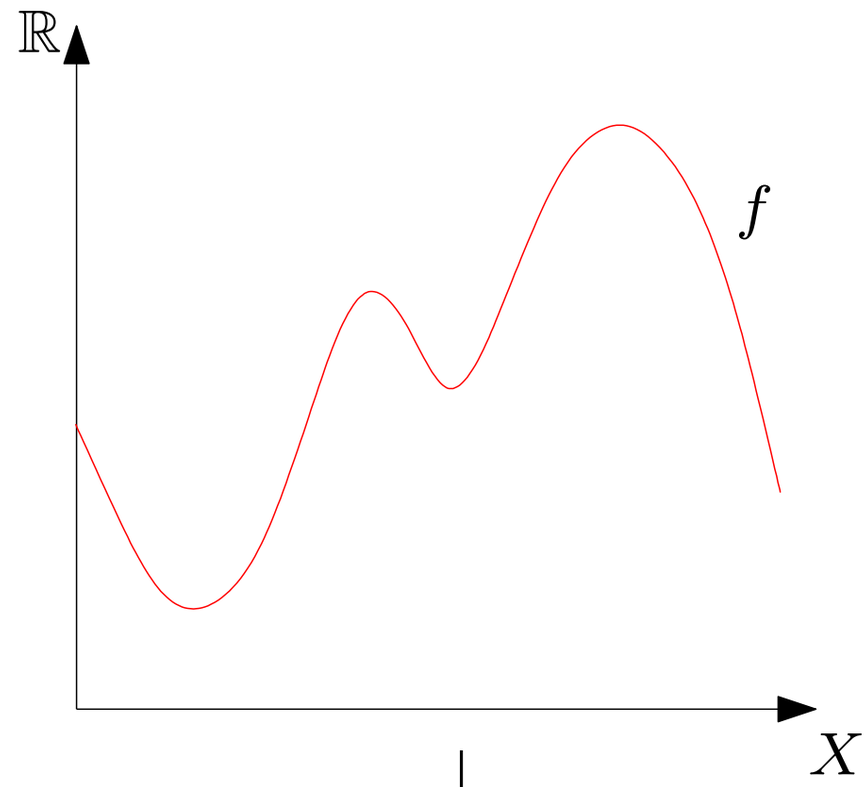
Ref: S.O. *Persistence Theory: from Quiver Representations to Data Analysis*. AMS Mathematical Surveys and Monographs, number 209, Dec. 2015.

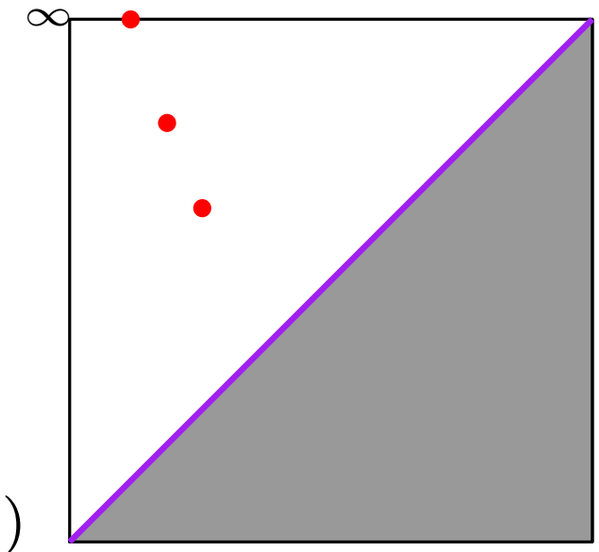
Topological Persistence (in a nutshell)

X topological space

$f : X \to \mathbb{R}$

$\mathrm{Dg}\ f$





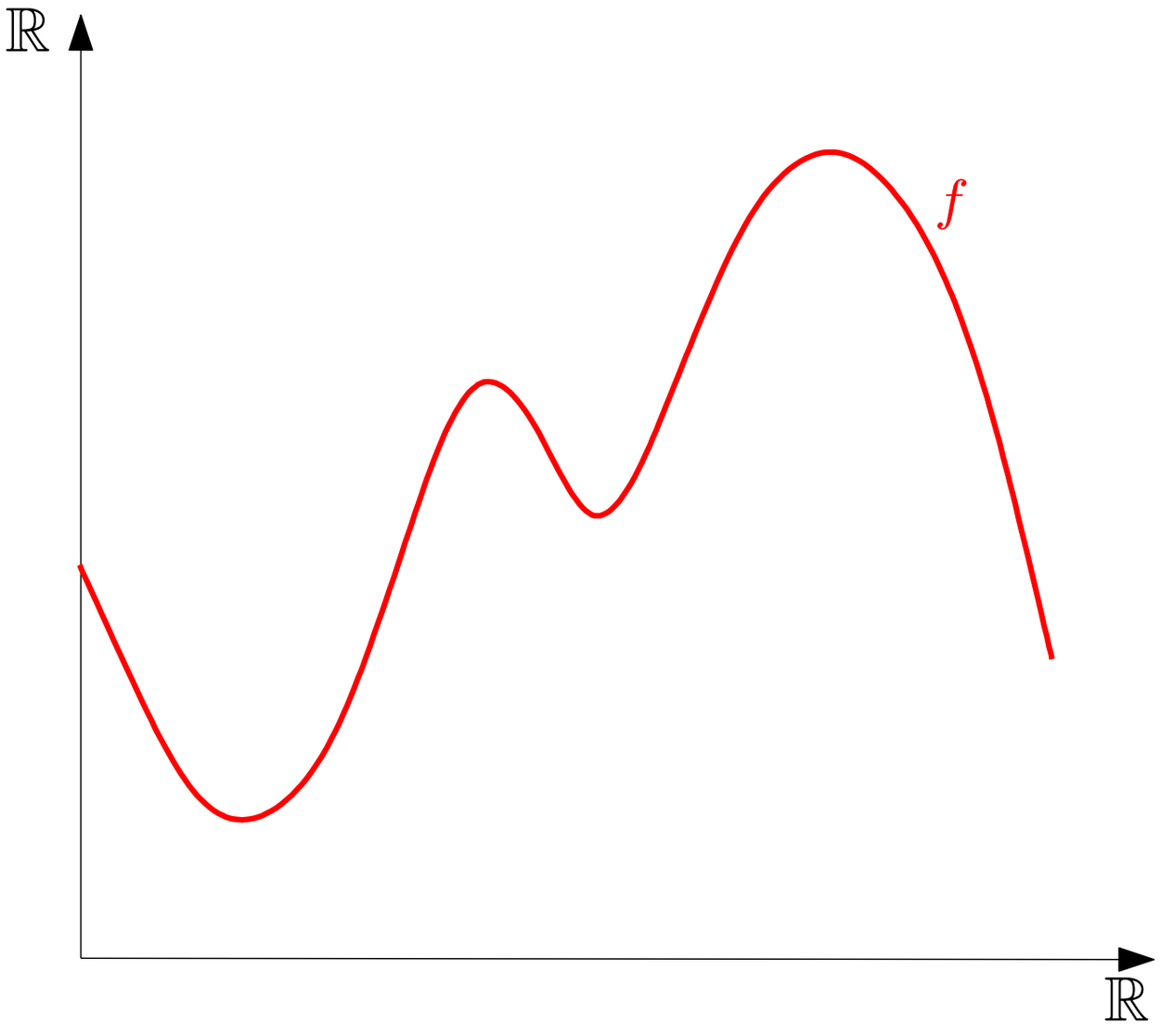
signature: *persistence diagram*

encodes the topological structure of the pair (X, f)

Topological Persistence (in a nutshell)

Inside the black box:

- Nested family (*filtration*) of sublevel-sets $f^{-1}((-\infty, \alpha])$ for α ranging over $\mathbb{R}$
- Track the evolution of the topology throughout the family



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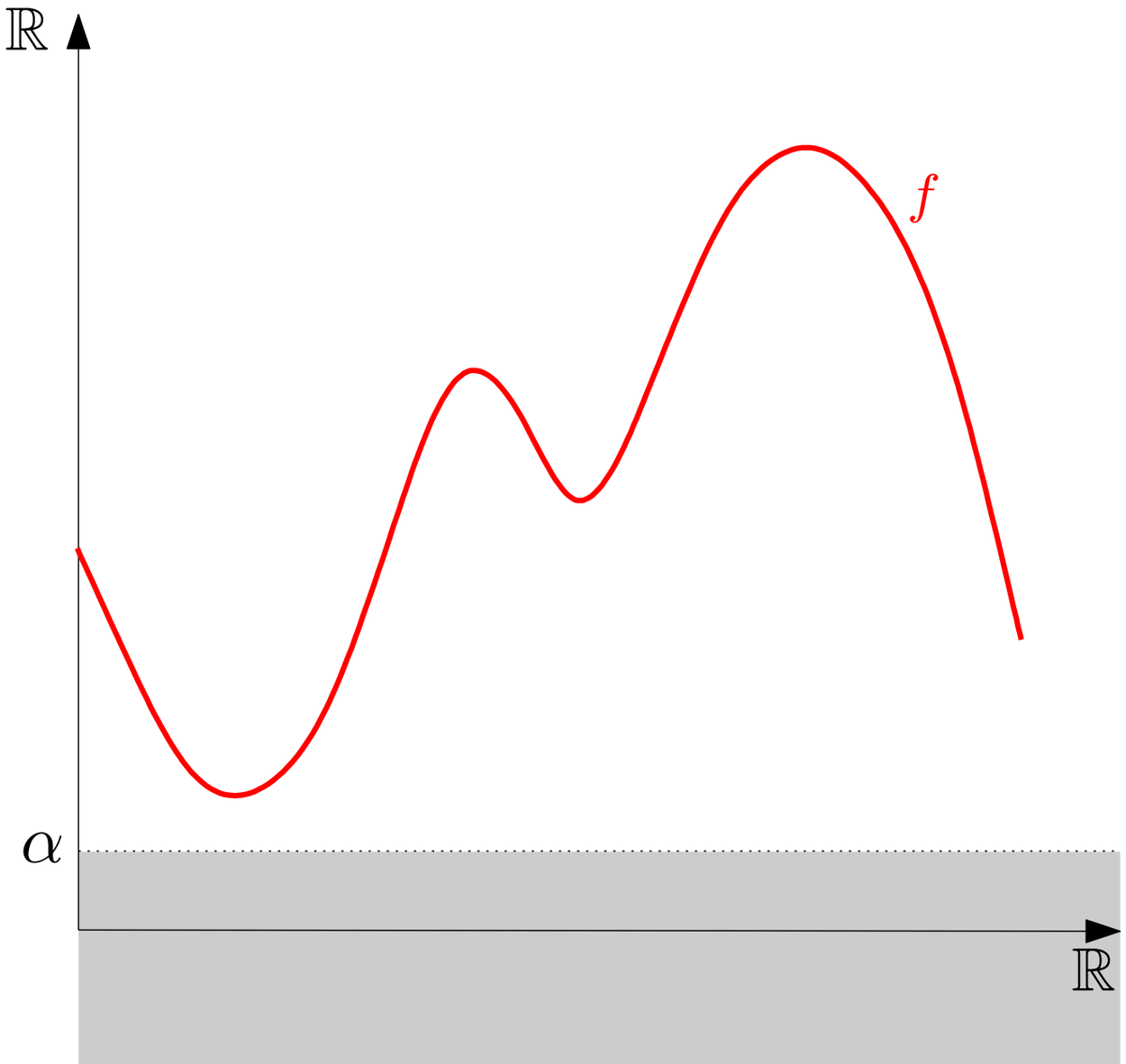

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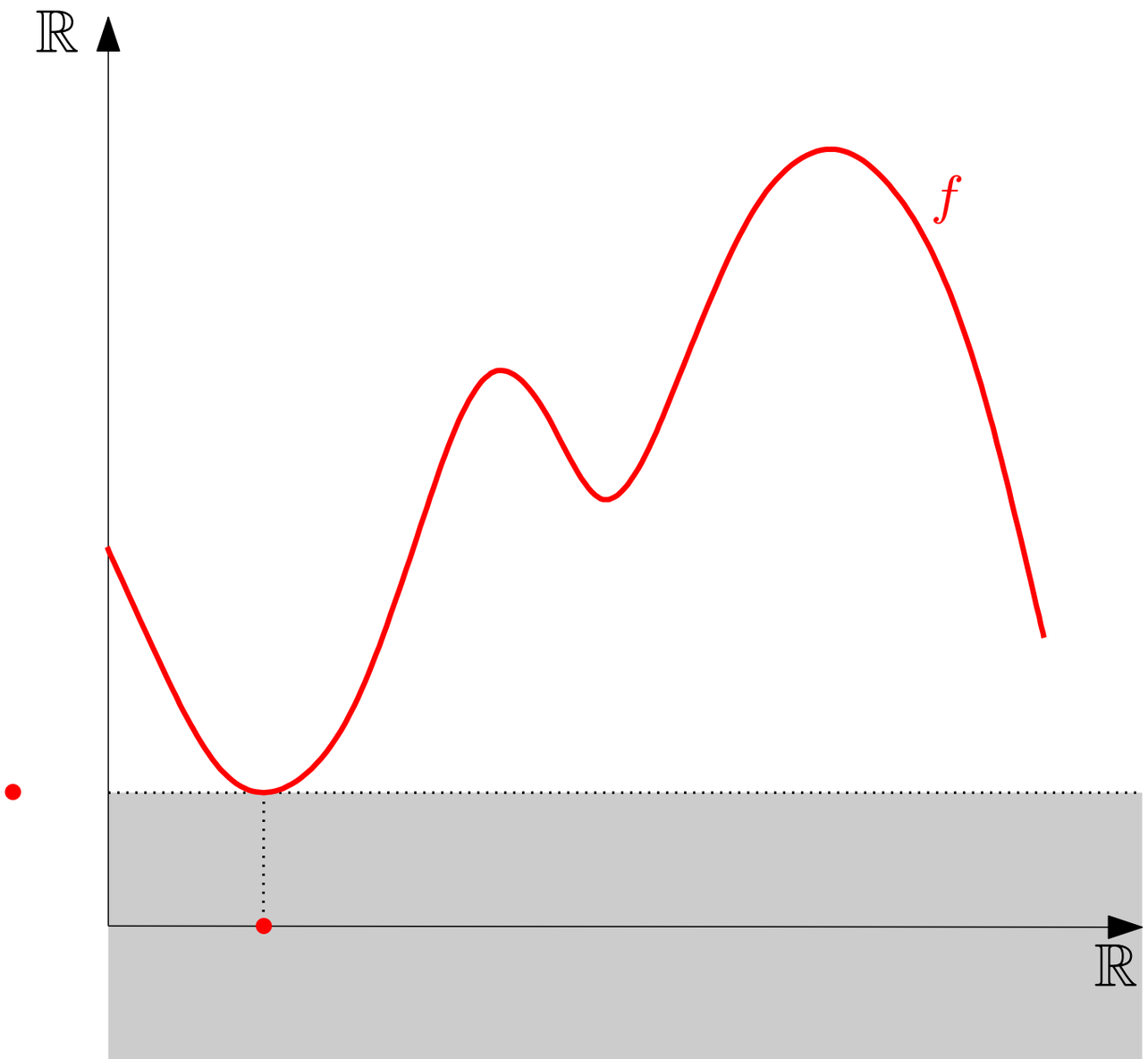

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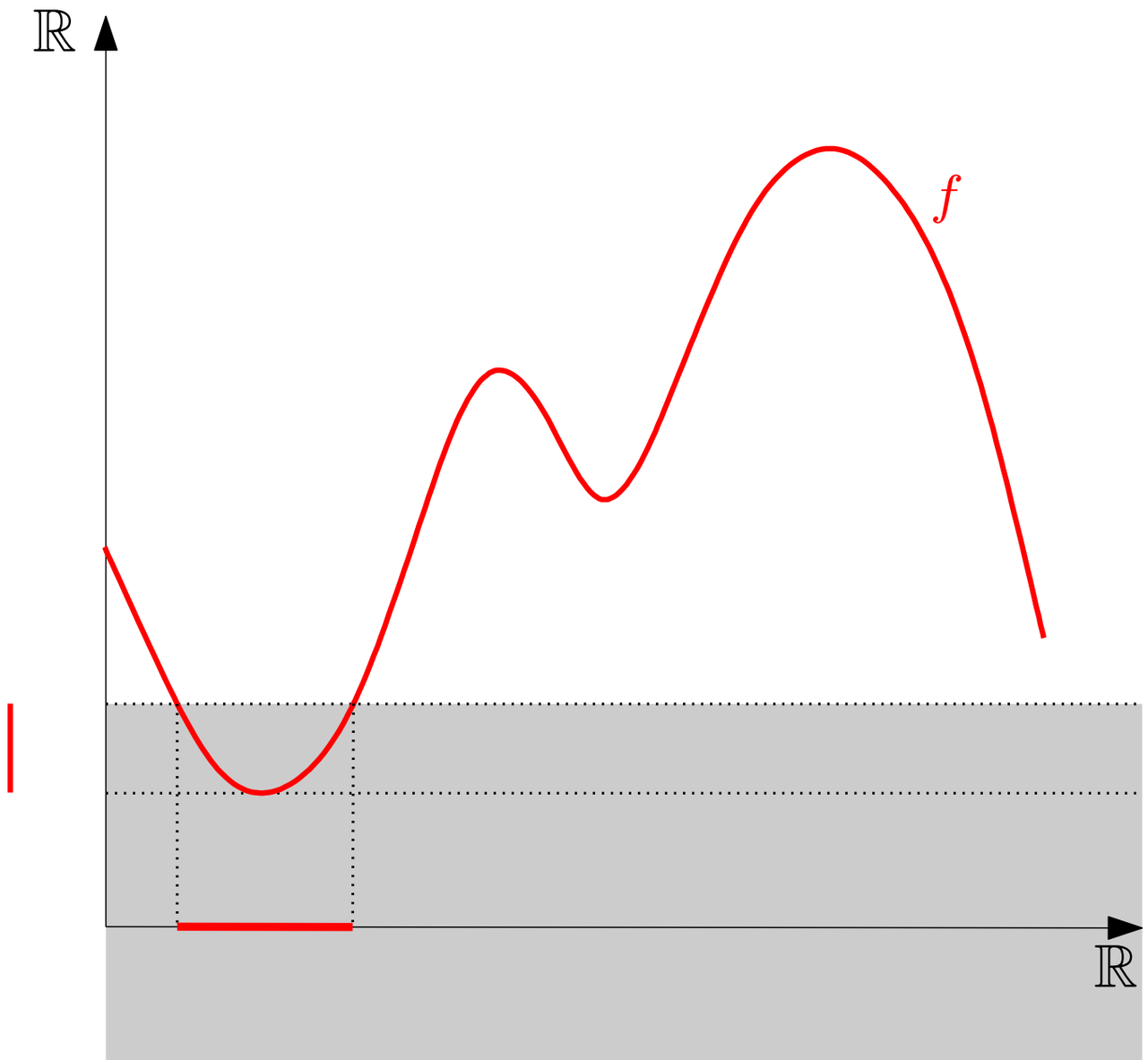

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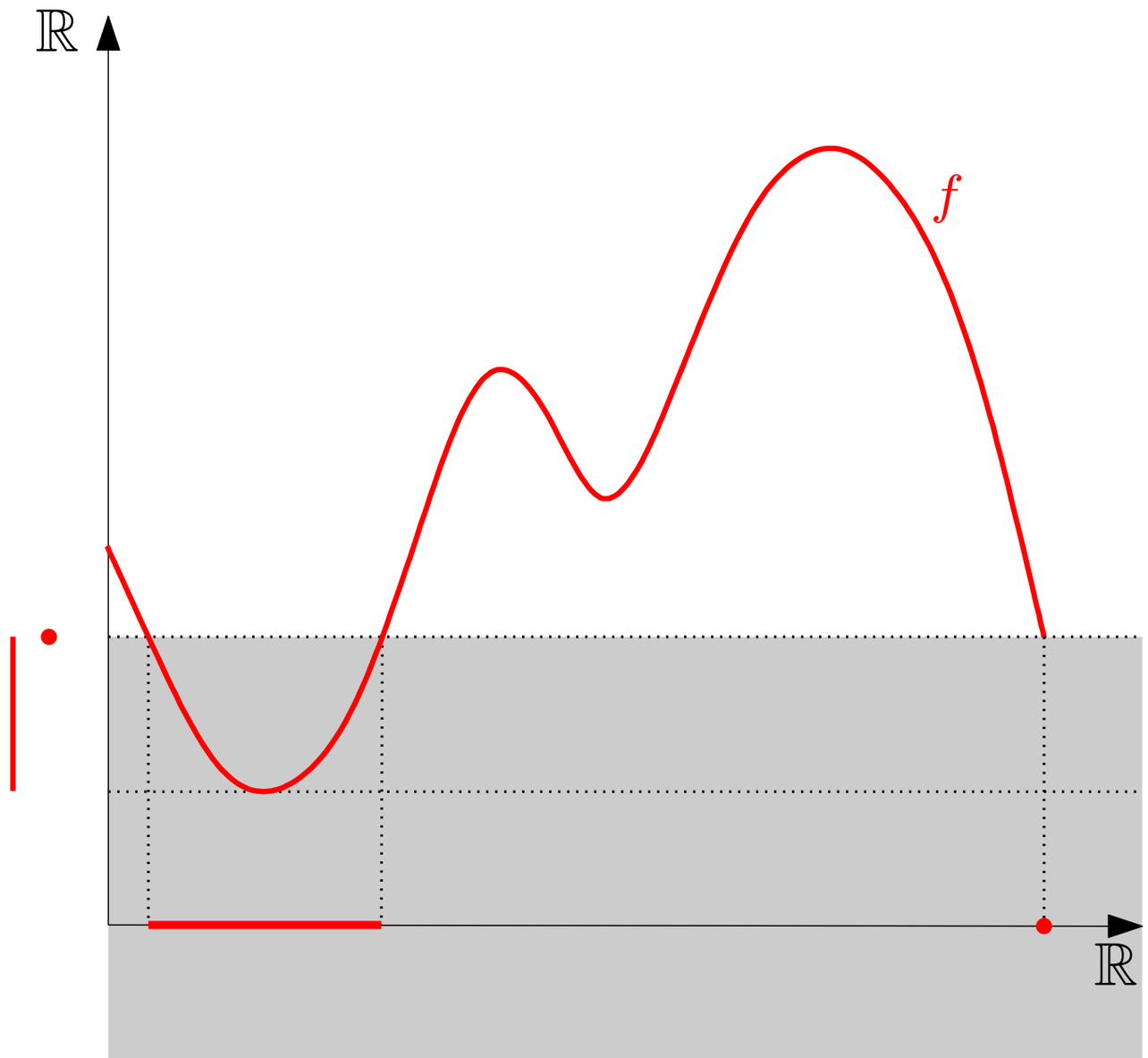

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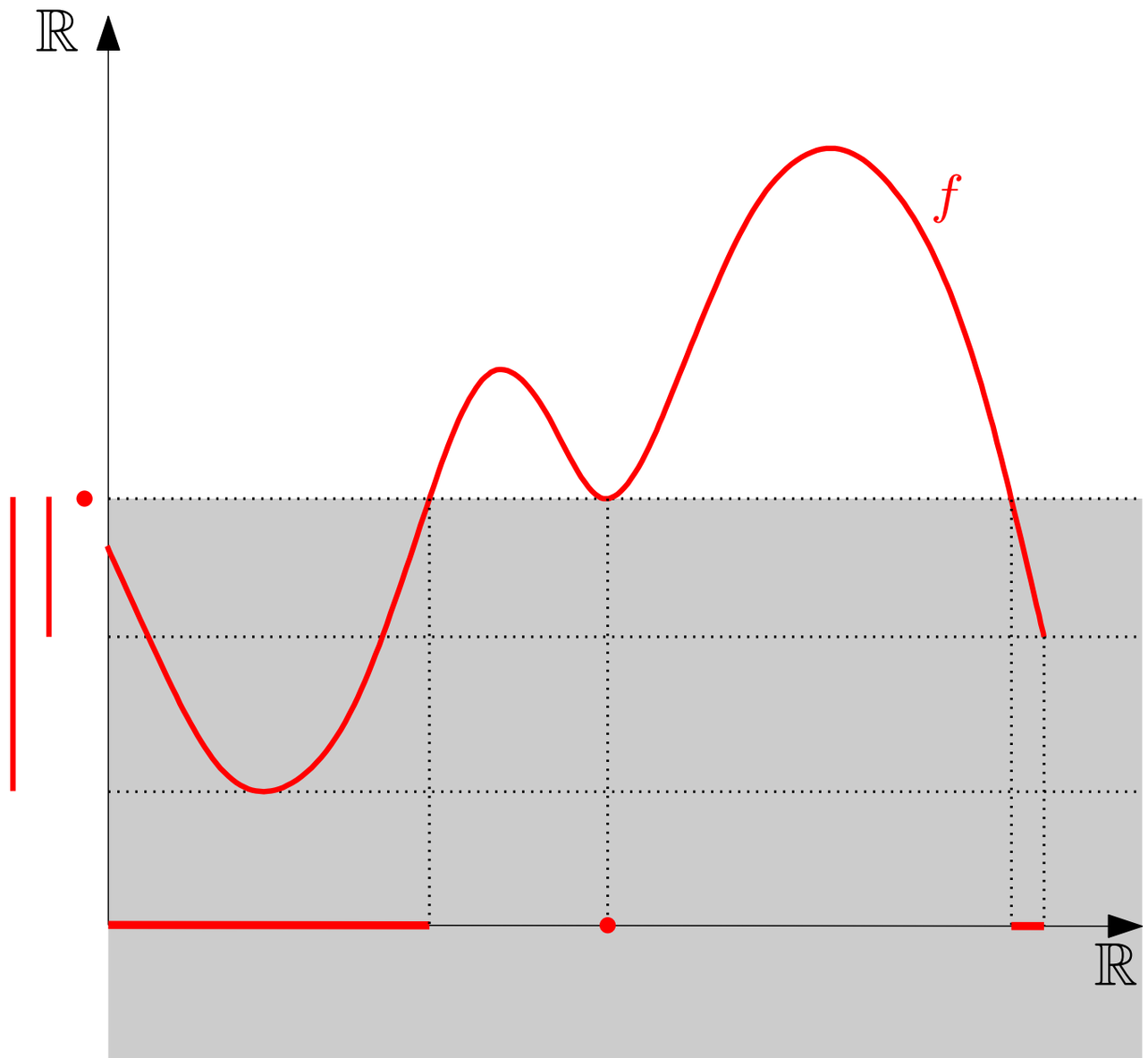

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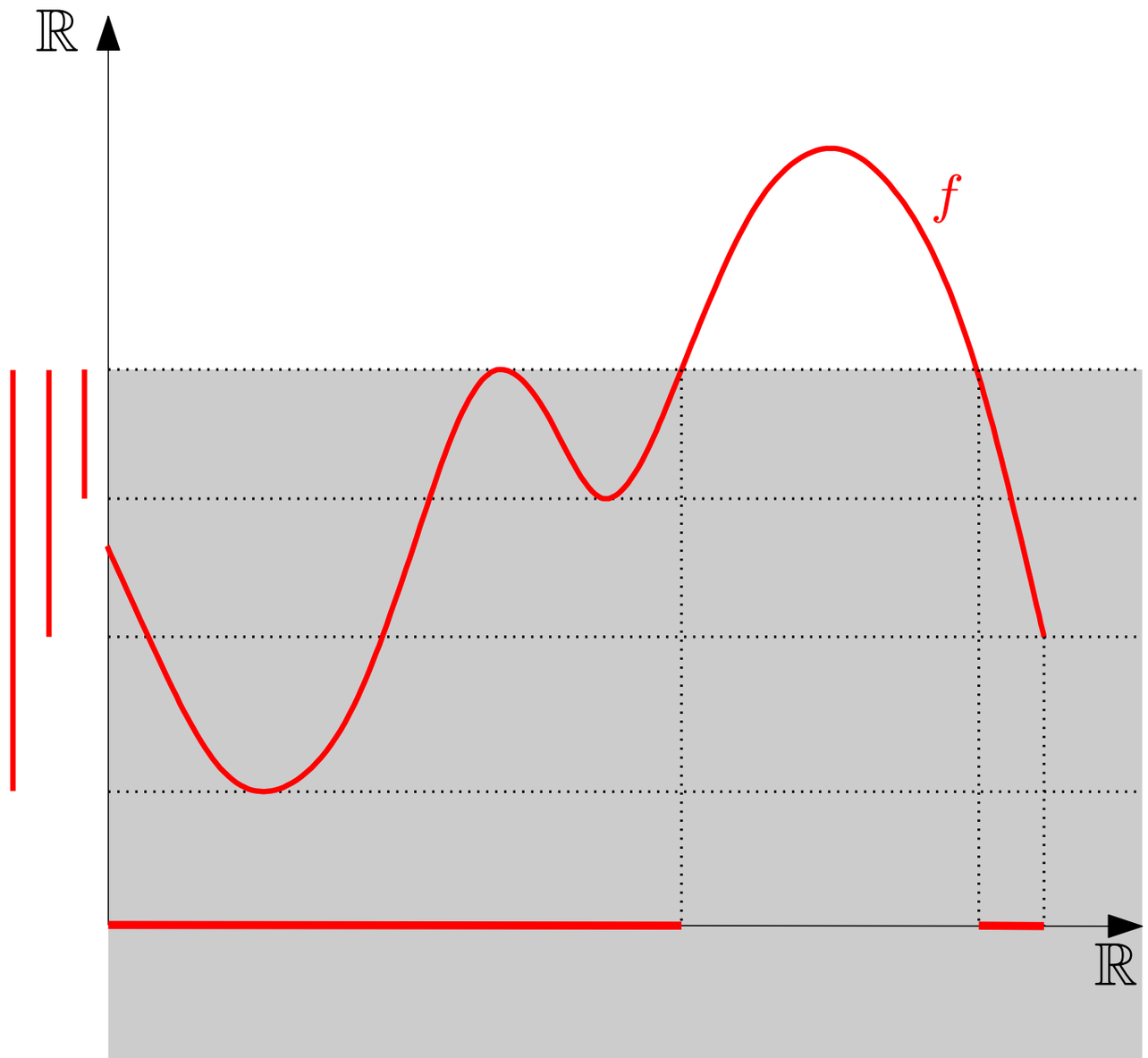

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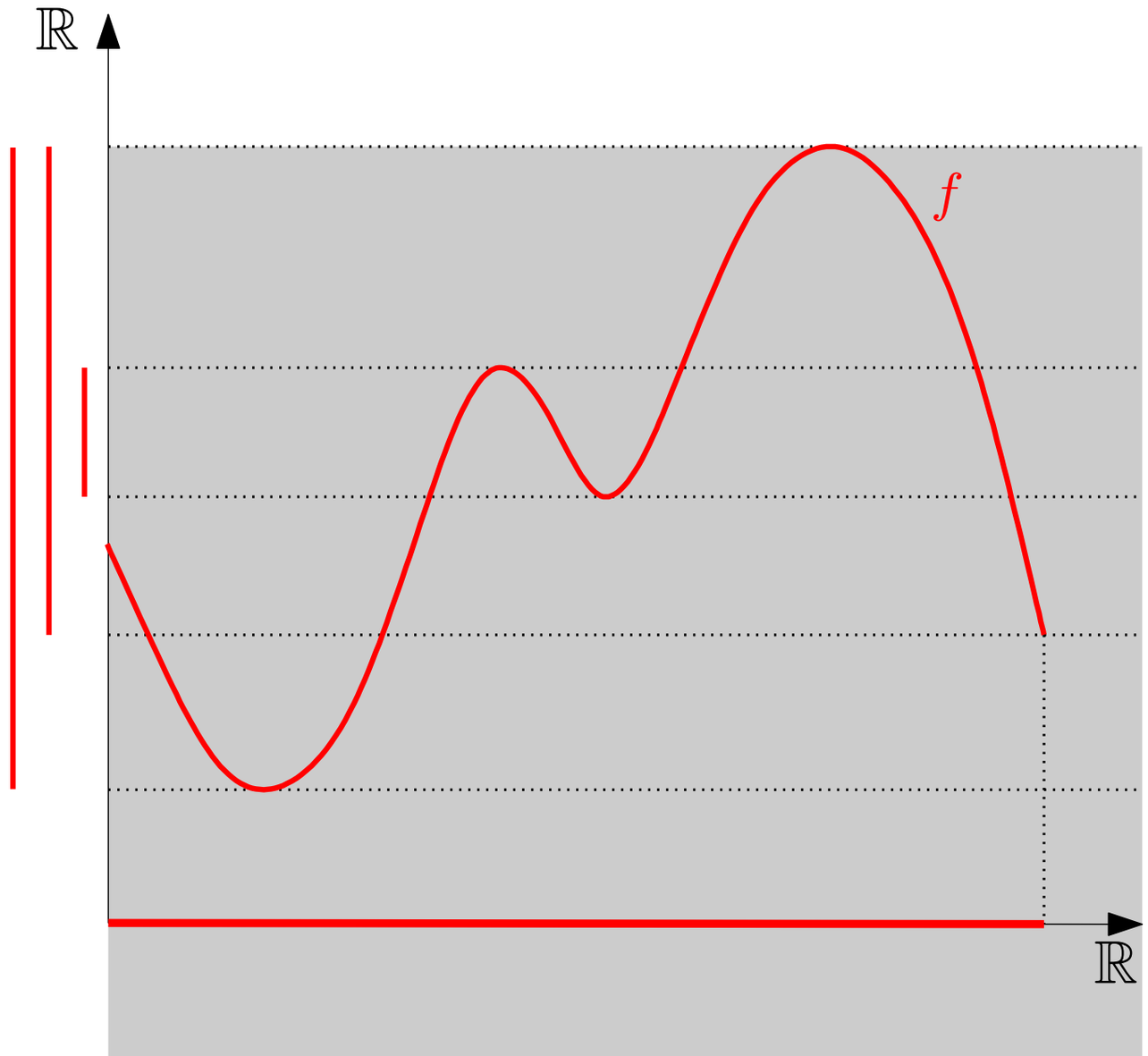

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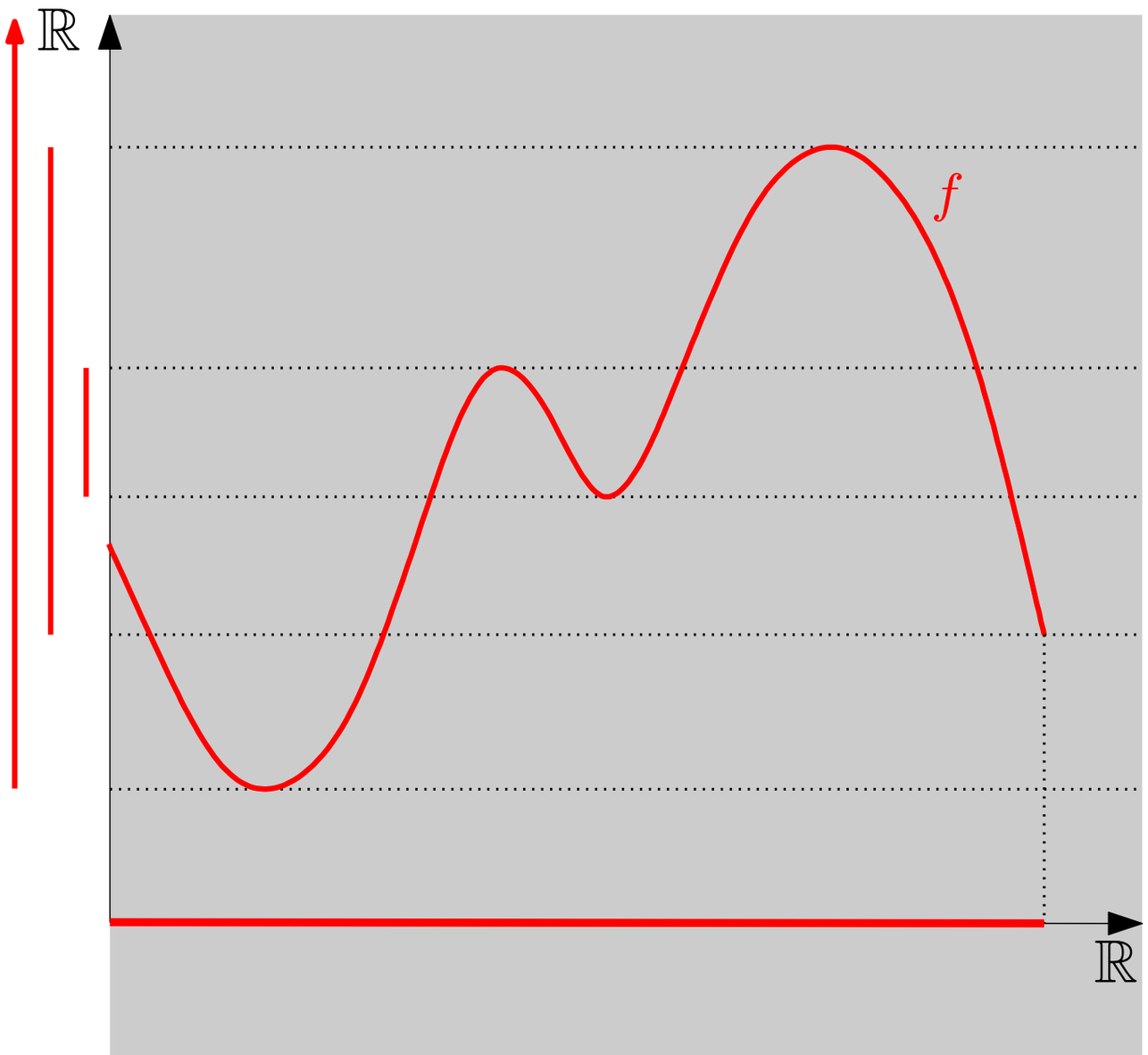
- Nested family (*filtration*) of sublevel-sets $f^{-1}((-\infty, \alpha])$ for α ranging over $\mathbb{R}$
- Track the evolution of the topology throughout the family
- Finite set of intervals (barcode) encodes births/deaths of topological features



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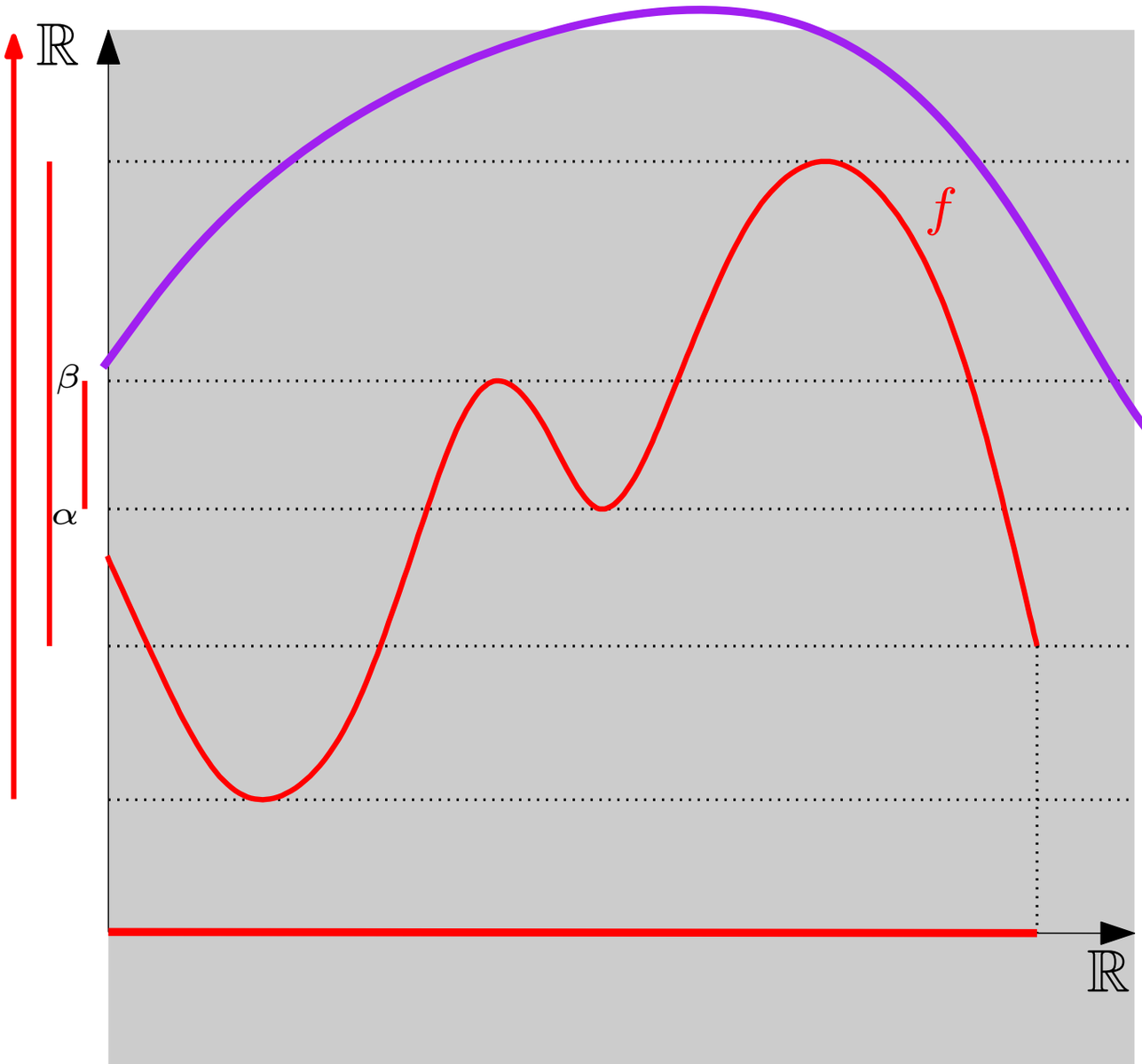
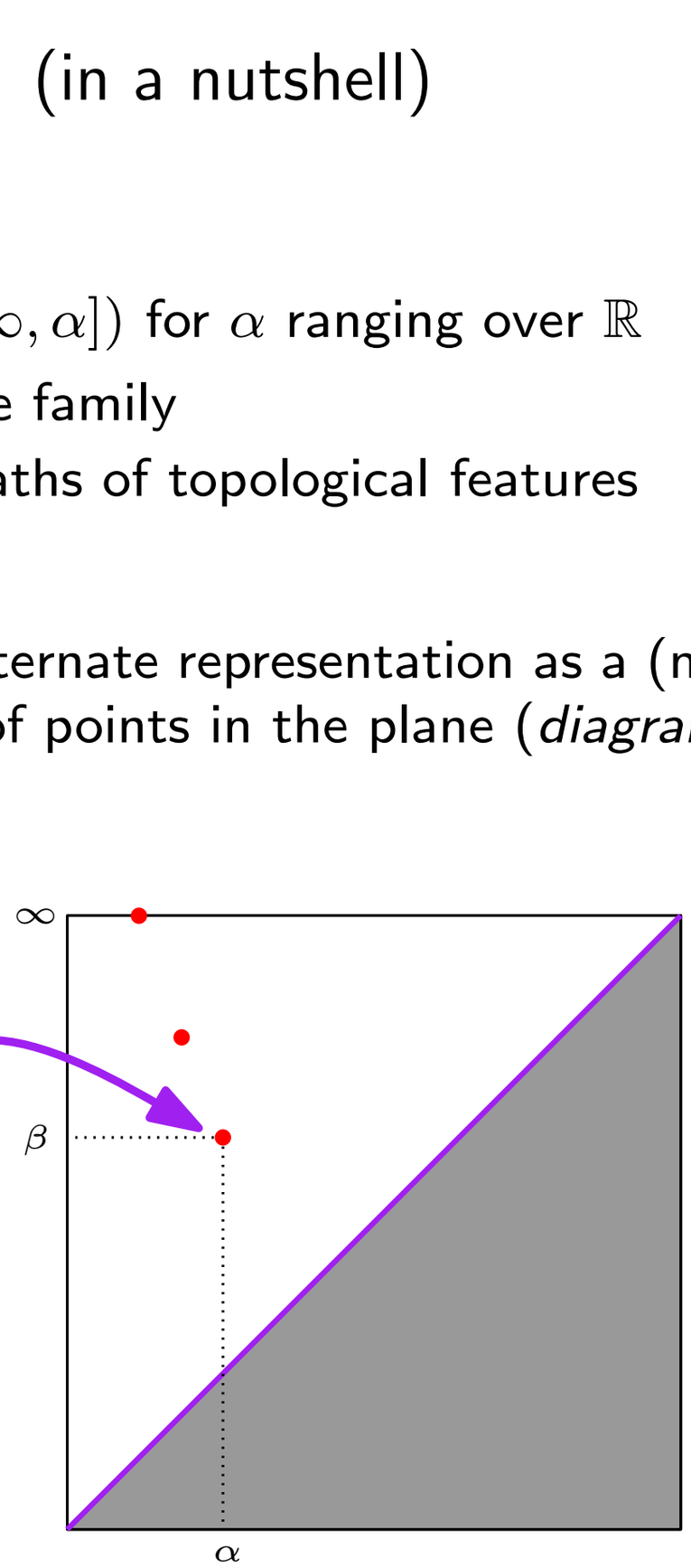


- Alternate representation as a (multi-) set of points in the plane (*diagram*).



Topological Persistence (in a nutshell)

3 pillars:

1. Decomposition theorems (existence of barcodes / diagrams)

2. algorithm (computation of barcodes / diagrams)

3. stability theorem (use of barcodes as signatures in applications)

Course Outline

Monday:

11h-12h30: persistence modules + decomposition theorems

16h-17h30: algorithm + stability

Tuesday:

9h-10h30: topological inference

14h-15h30: topological signatures I (global signatures)

Wednesday:

11h-12h30: topological signatures II (local signatures + kernels)

14h-15h30: TP

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- bring your laptop
- install R and TDA package (see *Set Up* section in TP)

http://geometrica.saclay.inria.fr/team/Steve.Oudot/courses/EMA/TP/

Thursday:

9h-10h30: clustering I (ToMATo)

11h-12h30: clustering II (Mapper)