3D Scale-space for Point Sets

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Introduction

- Object acquisition by a high precision laser scanner
- **Goal**: accurate analysis of the object’s geometry
- Application: automatic objects scanning by closed loop data analysis
**Triangulation laser scanner:** triangle formed by the camera optic center, the laser emitter and the impact point The result given by the scanner is a list of unoriented 3D points
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• The initial dataset is a set of positions with no connectivity information
• Additional constraint: holes should be detected but not filled, information should be given on the initial raw dataset
• Level sets methods ([HDD⁺92],[Kaz05],[KBH06]...) cannot be used
• **It leads us to a scale space approach**
Outline

1 Previous Work

2 3D Scale Space
   - Analogy in 2D image processing
   - 3D equivalent evolution equation

3 Application: Mesh reconstruction Problem
Multiscale Methods

- Most multiscale approaches are defined on meshes
- In [PKG06], a curvature motion is defined and a back projecting operator is associated

![Multiscale Decomposition and Reconstruction Diagram](image-url)
Triangulation Methods

Triangulation of a set of points by level sets method ([HDD+92],[KBH06]) (approximating methods):
A signed distance function to the surface is defined and its 0 level set is sampled by the marching cube algorithm for example.
Triangulation by Voronoi/Delaunay methods (interpolating methods)
Ball Pivoting Algorithm ([BMR\textsuperscript{+}99]): A triangle between three points is created if the $r$ radius sphere going through those three points does not contain any other point.
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Analogy in 2D image processing

Evolve an image by a PDE (e.g. heat equation), perform treatment at a low resolution and propagate the results to the original image.
3D equivalent evolution equation

Let $p$ be a point of a data set $\mathcal{M}$ and denote by $B_r(p)$ the set of all points $q$ in $\mathcal{M}$ such that $\|p - q\| < r$.

**Theorem**

In the local intrinsic coordinate system of a continuous and smooth two-dimensional manifold $\mathcal{M}$, for $p \in \mathcal{M}$ the projection $p'$ of $p$ on the local regression plane has coordinates $x_{p'} = 0$, $y_{p'} = 0$ and $z_{p'} = \frac{Hr^2}{4} + o(r^2)$, where $H = \frac{k_1 + k_2}{2}$ is the surface mean curvature at $p$ and $k_1$, $k_2$ the surface principal curvatures at $p$. 
Consequence

- \( H \approx \frac{4\langle p-p', \hat{n}(p) \rangle}{r^2} \)
- It is a very stable estimate since it relies on order 1 approximation
- This yields our scale space operator:

**Theorem**

*Let \( T_r \) be the operator defined on the surface \( M \) transforming each point \( p \) into its projection on the local regression plane. Then*

\[
T_r(p) - p = -\frac{Hr^2}{4} \hat{n}(p) + o(r^2). \tag{1}
\]

*Thus, this operator is tangent to the mean curvature motion.*
Estimated Curvature on a perfect torus

Figure: left: theoretical curvature, right: scale-space estimated curvature
Estimated Curvature on various point sets
Visualization of scale space effects

- A good way of seeing the evolution of a surface defined by a point cloud is to watch the evolution of level sets and especially positive and negative curvature level sets.
- At first the level sets capture texture variations but when the surface is smoothed, is captures the object’s shape.
Visualization of scale space effects

Figure: figs 1-4: 4 iterations of scale space are applied, fig. 5: by reverse scale space points are moved back to their original positions
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3. Application: Mesh reconstruction Problem
For each point normal direction is known by neighborhood PCA
Choose a point in a flat area, pick one of the two possible orientations
Propagate orientation in neighborhoods
Algorithm

1. Iterate Projection Filter and keep a track at each step of the point displacement
2. Mesh the resulting samples. The obtained mesh is singularity free;
3. Project the mesh back to the original points.
4. The result is an interpolating mesh which preserves textures and details.
synthetic 1D example

Initial points
synthetic 1D example

Initial points and their projections
synthetic 1D example

Projected points
synthetic 1D example

Resulting mesh of the projected points
synthetic 1D example

Back projected mesh of the initial points
**synthetic 1D example**

Top: same initial points with direct BPA triangulation

Bottom: same initial points with Poisson Reconstruction
Poisson Reconstruction of a car point set
Scale Space Meshing of a car point set
Original Object: 20 cm high tanagra
Coarse resolution Mesh (after projection iterations)
Mesh obtained at a high resolution (back-projected)
3D Scale-space for Point Sets

Application: Mesh reconstruction Problem
Comparison with other methods
Figure: Direct meshing [BMR⁺ 99]
Figure: Poisson Reconstruction [KBH06]
Figure: Scale Space Meshing
Comparison between direct meshing...
...and scale space meshing


Discussion: radius choice

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- $r$ should be large enough to guarantee stability but small enough to guarantee mathematical consistency
- In practice we found that $n = 20$ is enough
- Sampling irregularities: in areas where the density is too low, if the distance between points is more than $2r$ no triangulation is given
The mesh allows to:

- detect curvature level lines as polylines
- detect hole borders as polylines
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- This mesh interpolates points and preserves well details and textures.
- Future Work will focus on using the scale-space framework to deal with point cloud registration, crest line detection.


Michael Kazhdan, Reconstruction of solid models from oriented point sets, SGP '05: Proceedings of the Third Eurographics Symposium on Geometry processing (Aire-la-Ville, Switzerland, Switzerland), Eurographics Association, 2005, p. 73.

Michael Kazhdan, Matthew Bolitho, and Hugues Hoppe, Poisson surface reconstruction, SGP '06: Proceedings of the fourth Eurographics Symposium on Geometry processing (Aire-la-Ville, Switzerland, Switzerland), Eurographics Association, 2006, pp. 61–70.
