

<http://www-sop.inria.fr/geometrica/collaborations/TGDA/index.html>

TGDA

Topological and Geometric Data Analysis

Geometric Computing group
Stanford University



Geometrica group
INRIA Saclay / Sophia



Official collaboration between research groups



- Geometric Computing group

(L. J. Guibas, 15 people)



- Geometrica group

(J-D Boissonnat, 25 people)



Official collaboration between research groups



- **Geometric Computing group**

(L. J. Guibas, 15 people)

- Applied Topology group

- ML, Robotics, Bio-computation...



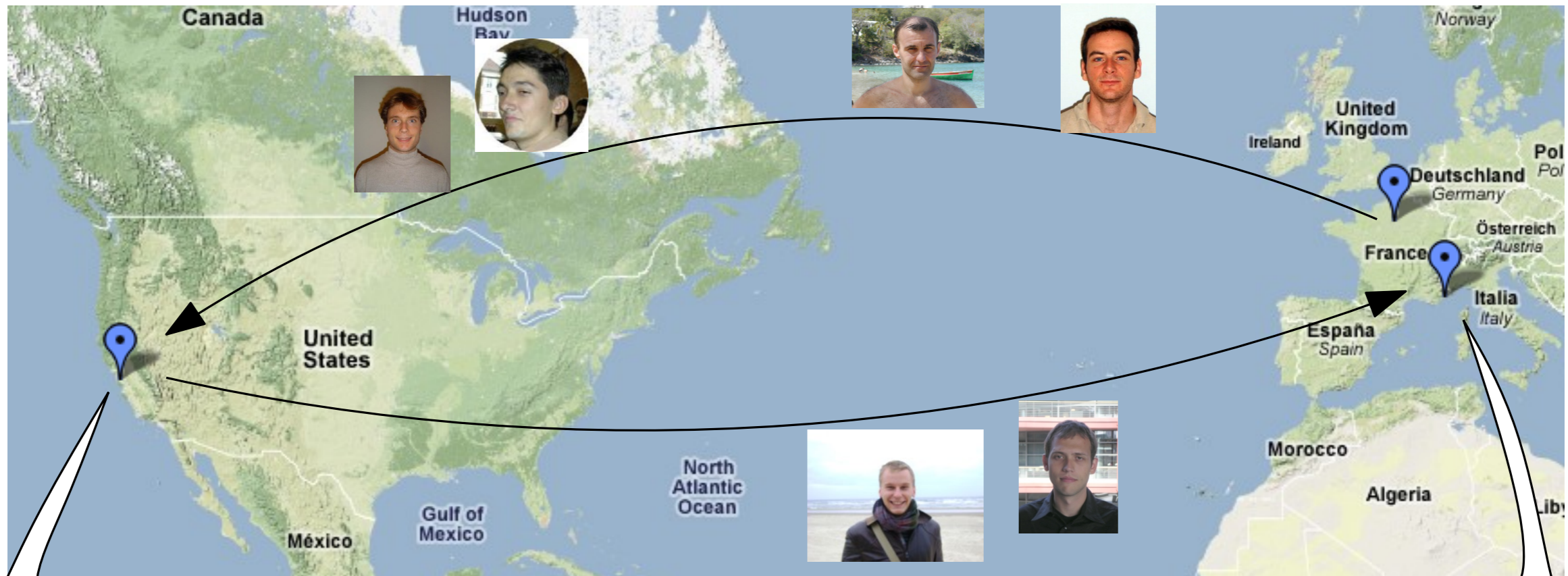
- **Geometrica group**

(J-D Boissonnat, 25 people)

- ABS, Select, Asap...



Exchanges (2008)



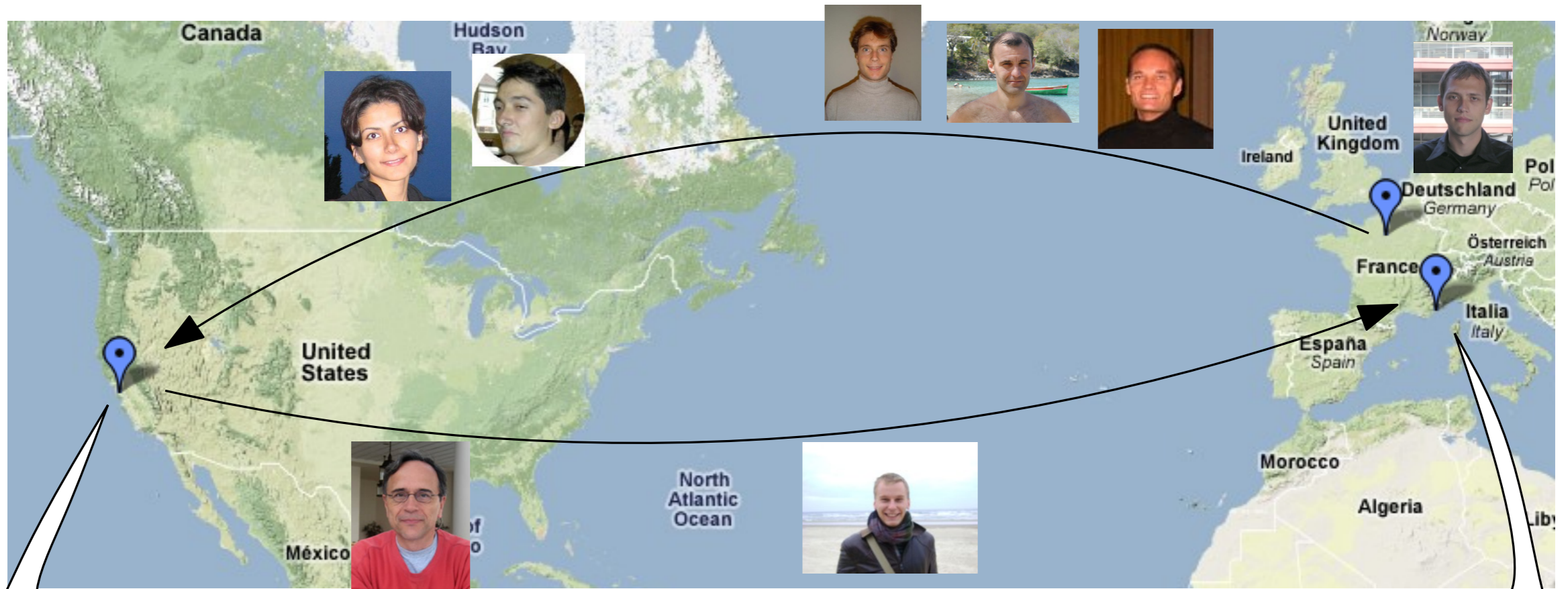
- Geometric Computing group

- long-term: M. Ovsjanikovs, P. Skraba

- Geometrica group

- long-term: Q. Mérigot, S. Oudot
- short-term: F. Chazal, D. Cohen-Steiner

Exchanges (2009)



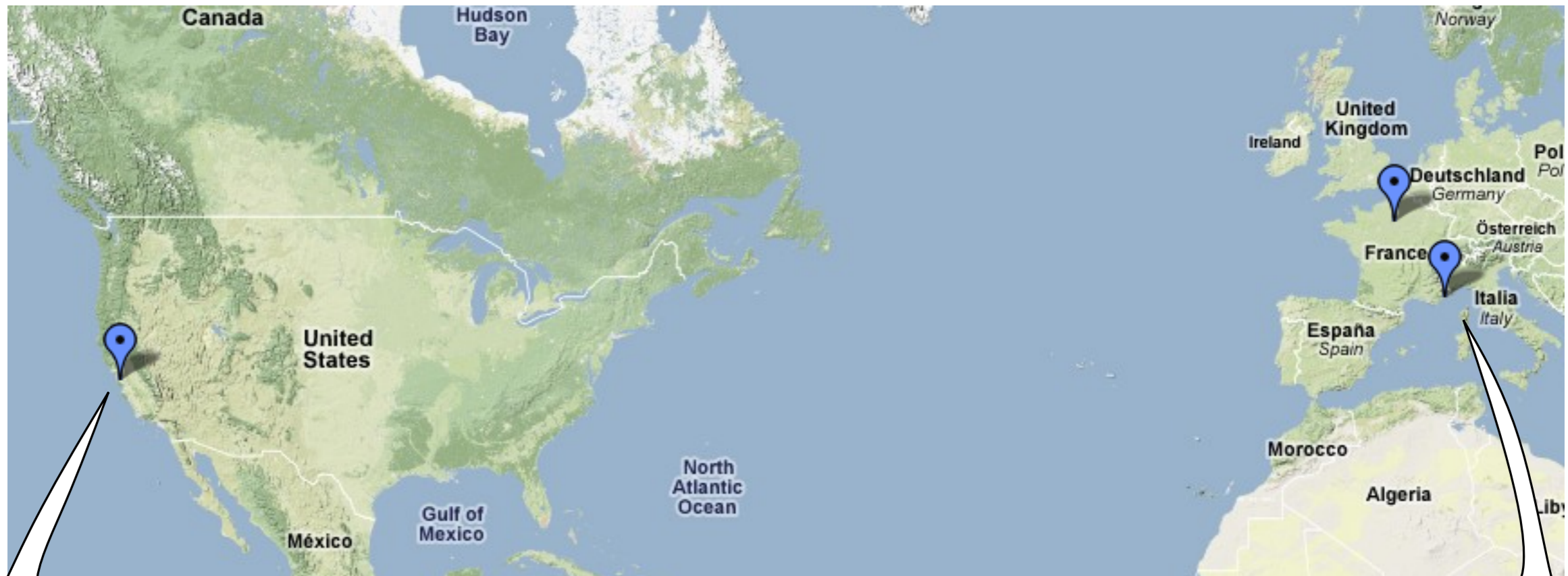
- Geometric Computing group

- long-term: M. Ovsjanikovs
- short-term: L. Guibas
- 2-year post-doc: P. Skraba

- Geometrica group

- long-term: Q. Mérigot, P. Memari
- short-term: F. Chazal,
J-D Boissonnat,
S. Oudot

Funding



- Geometric Computing group

18 kE in 2008 (NSF)

25 kE in 2009 (NSF + FFS)

■ ■ ■

- Geometrica group

20 kE in 2008 (EA)

20 kE in 2009 (EA)

■ ■ ■

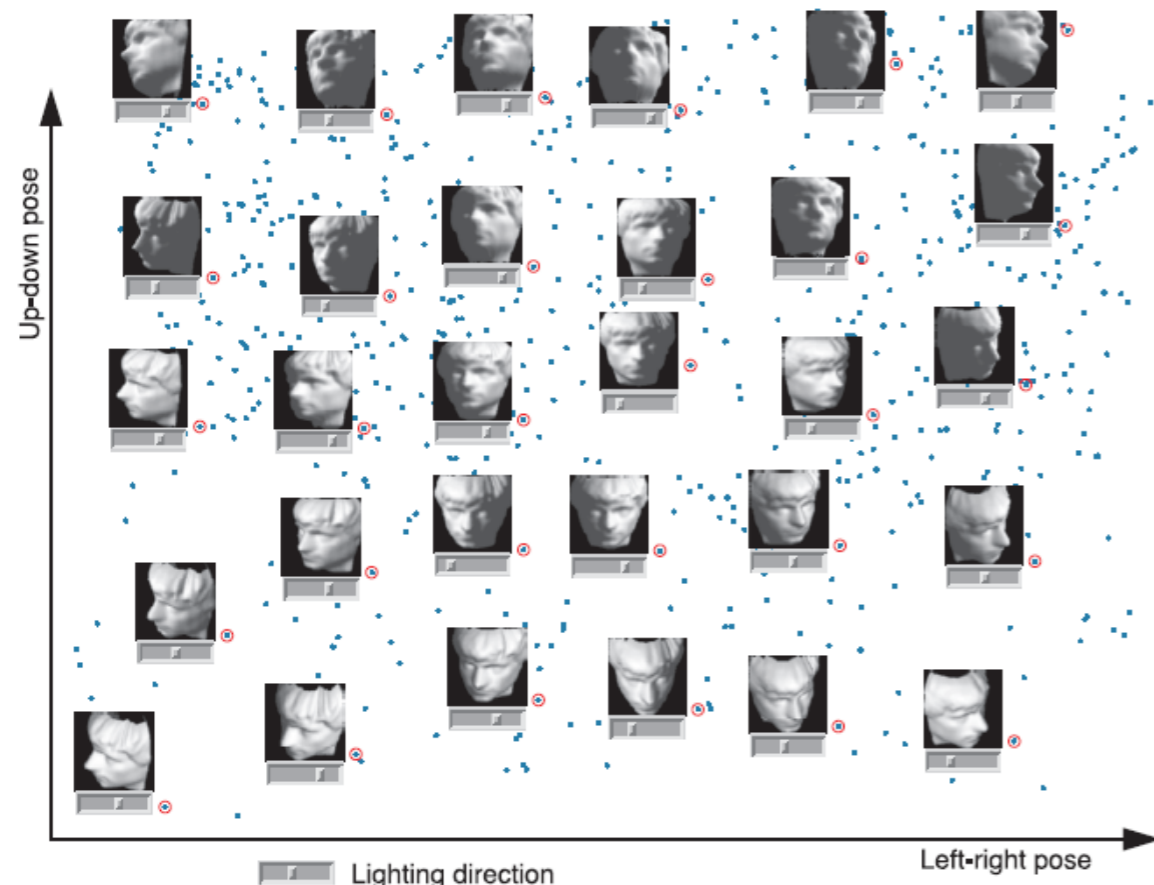
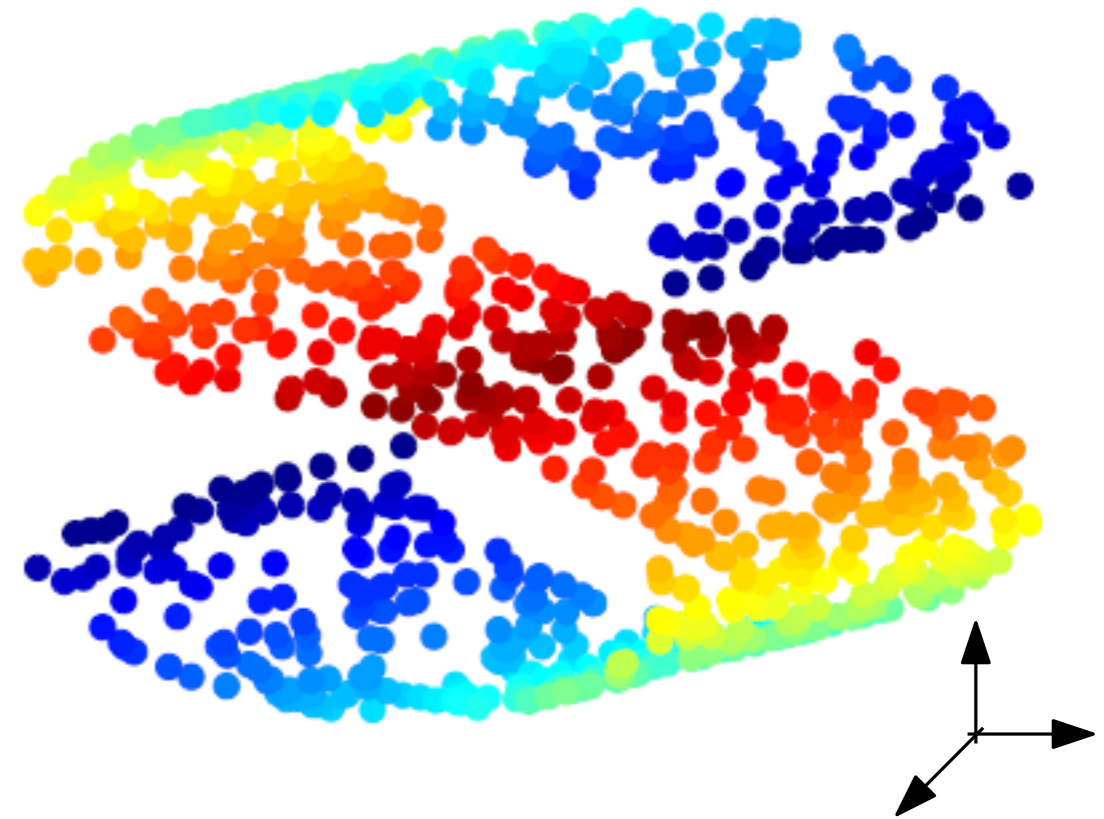
Main Scientific Goals

Input: a point cloud in a metric space.

Is there structure in the data?

Can we infer topological invariants?

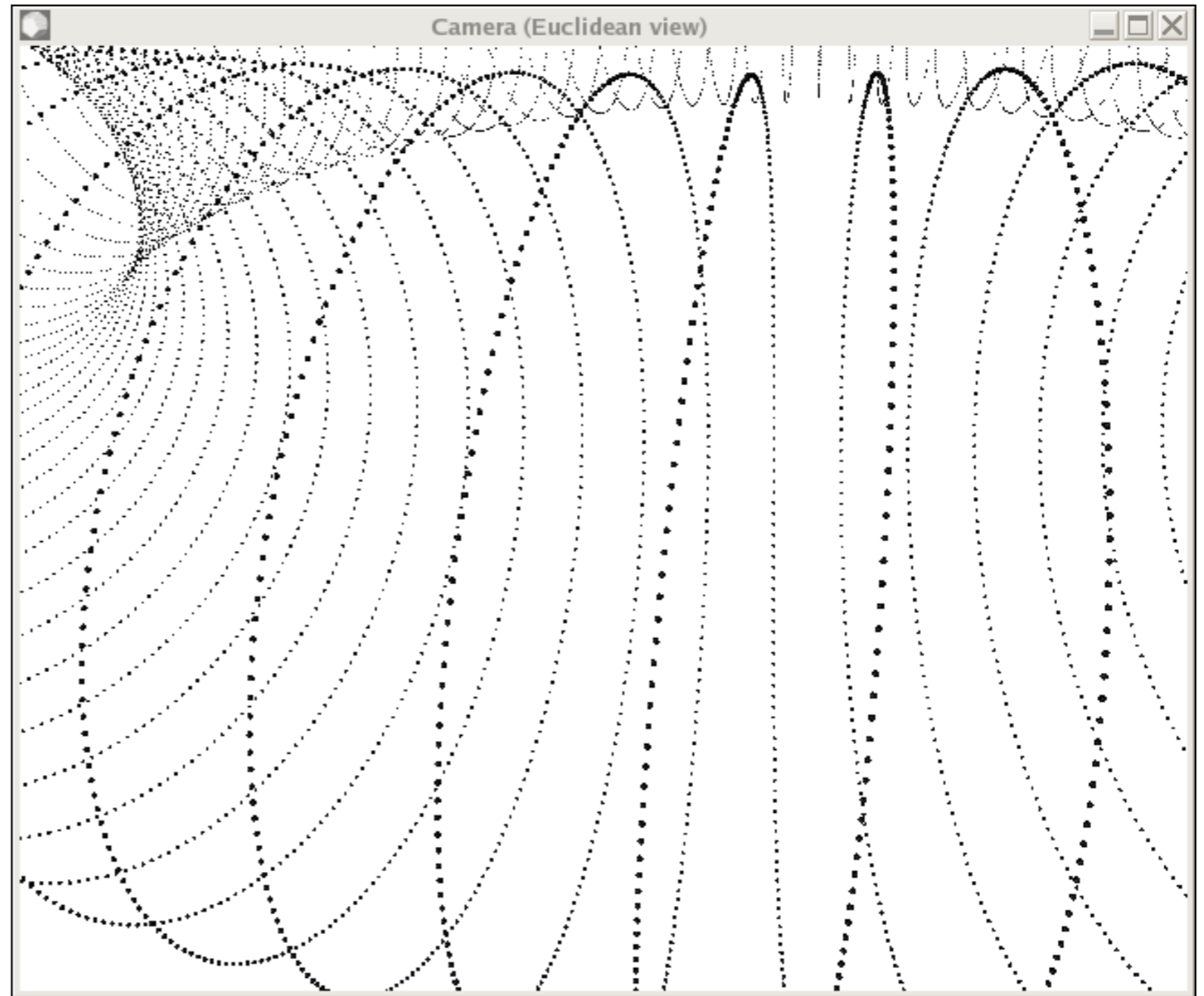
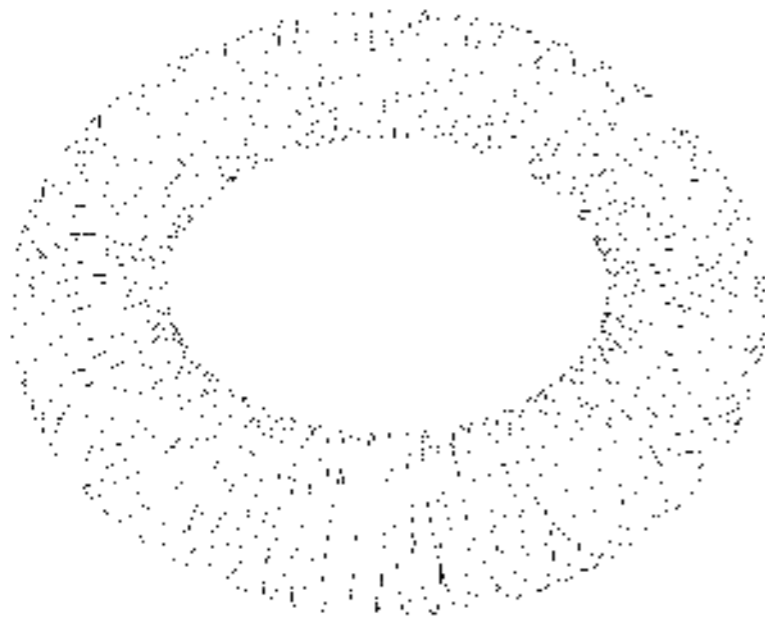
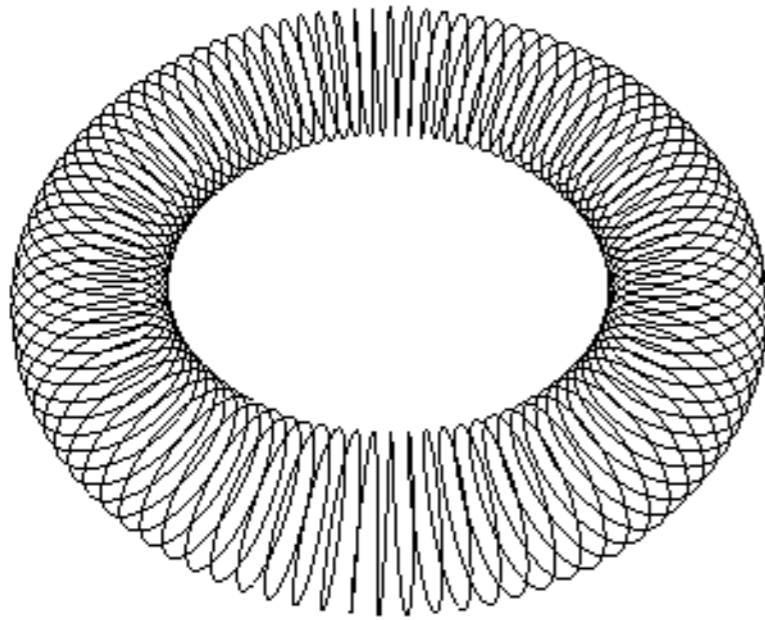
Can we approximate differential quantities?



Example: set of 4096-dimensional data points, representing 64x64 pixels images of a same object, seen under various lighting and camera angles. (from Isomap, *Science* 290).

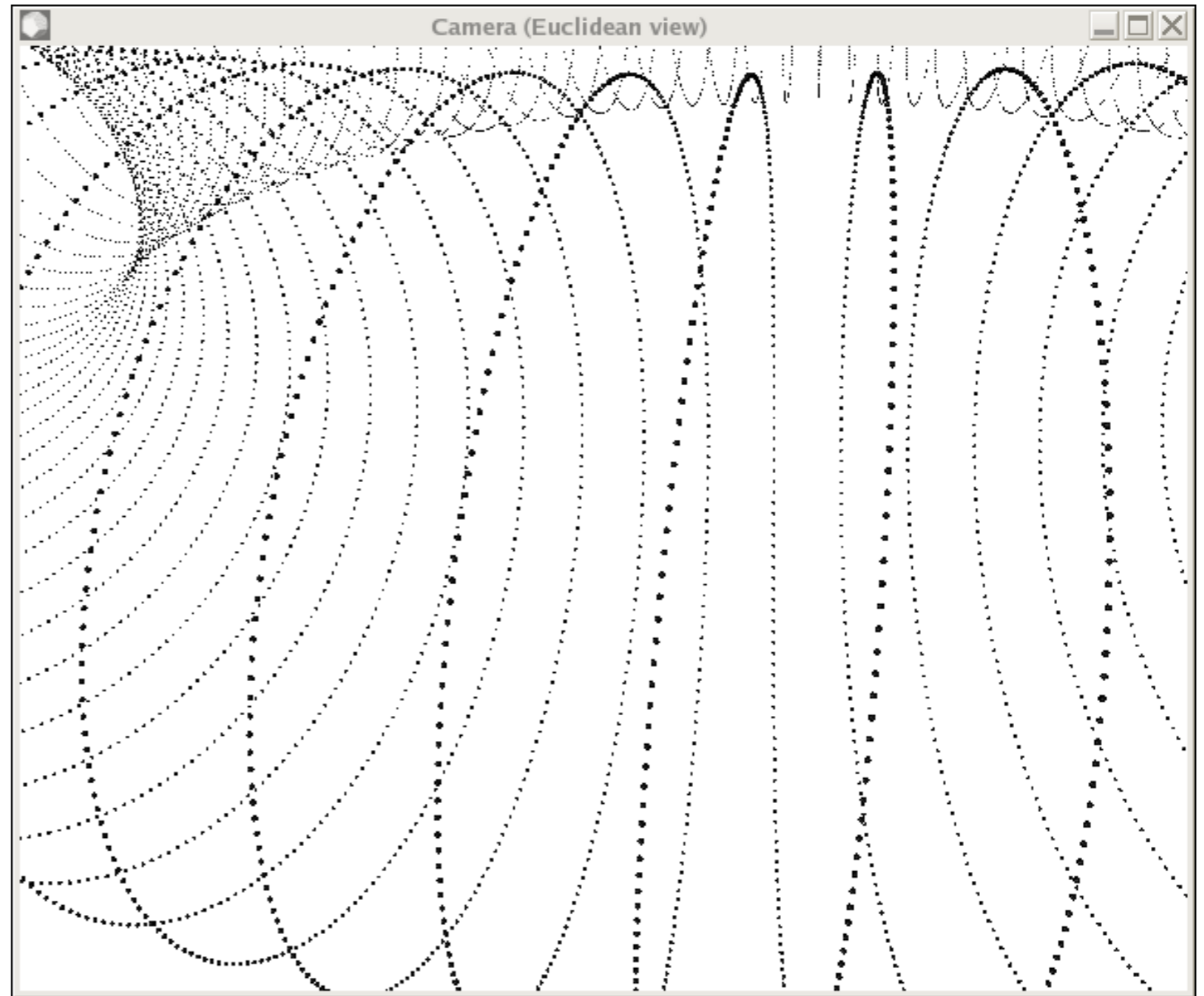
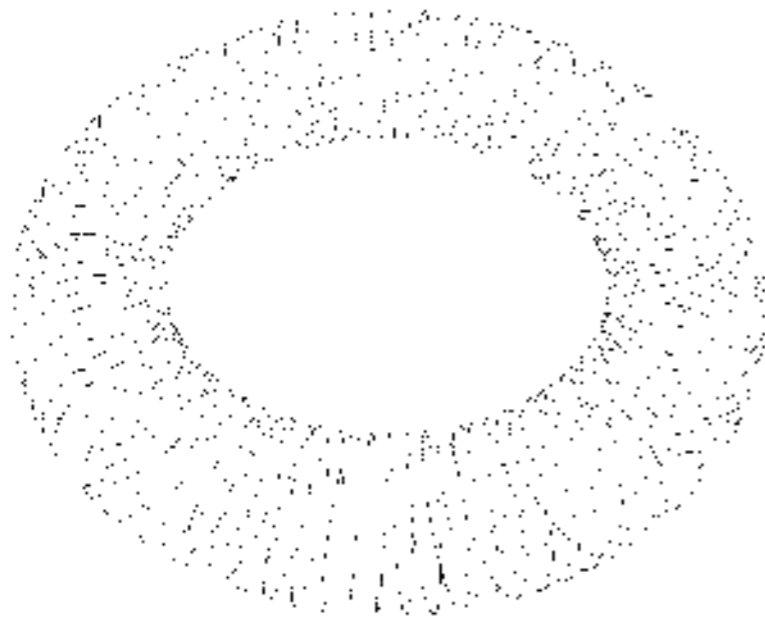
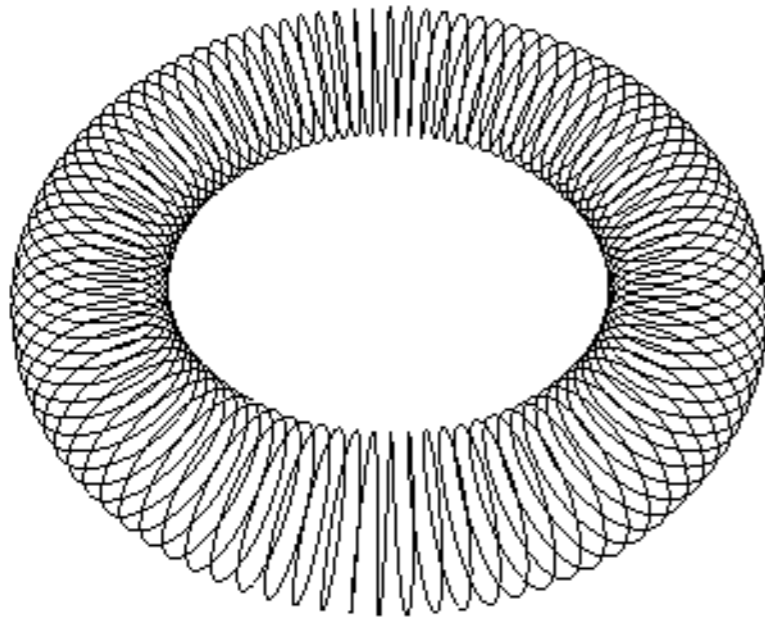
Theoretical Challenges

What is the underlying object?



Theoretical Challenges

What is the underlying object?



- perform multi-scale analysis (landmarking, topological persistence)

Algorithmic Challenges

Curse of dimensionality:

The ε -entropy ($\varepsilon < 1$) of a smooth k -dimensional submanifold M of Euclidean space \mathbb{R}^d is of the order of $-k \log(\varepsilon)$. In other words, for any finite set X s.t. $d_H(M, X) \leq \varepsilon$, $|X| = \Omega((\frac{1}{\varepsilon})^k)$.

Algorithmic Challenges

Curse of dimensionality:

The ε -entropy ($\varepsilon < 1$) of a smooth k -dimensional submanifold M of Euclidean space \mathbb{R}^d is of the order of $-k \log(\varepsilon)$. In other words, for any finite set X s.t. $d_H(M, X) \leq \varepsilon$, $|X| = \Omega((\frac{1}{\varepsilon})^k)$.

- assume high co-dimension: $1 \lesssim k \ll d$

Double curse:

For a finite set of points on a k -submanifold of \mathbb{R}^d , classical data structures from computational geometry and topology (Čech complex, Delaunay triangulation, α -shape) scale up exponentially with d , not k .

Algorithmic Challenges

Curse of dimensionality:

The ε -entropy ($\varepsilon < 1$) of a smooth k -dimensional submanifold M of Euclidean space \mathbb{R}^d is of the order of $-k \log(\varepsilon)$. In other words, for any finite set X s.t. $d_H(M, X) \leq \varepsilon$, $|X| = \Omega((\frac{1}{\varepsilon})^k)$.

- assume high co-dimension: $1 \lesssim k \ll d$

Double curse:

For a finite set of points on a k -submanifold of \mathbb{R}^d , classical data structures from computational geometry and topology (Čech complex, Delaunay triangulation, α -shape) scale up exponentially with d , not k .

- Build lightweight data structures (Rips complex, witness complex)

Theoretical Tools

Delaunay

- restricted Delaunay
- ε -sampling theory
- α -shape
- Witness complex

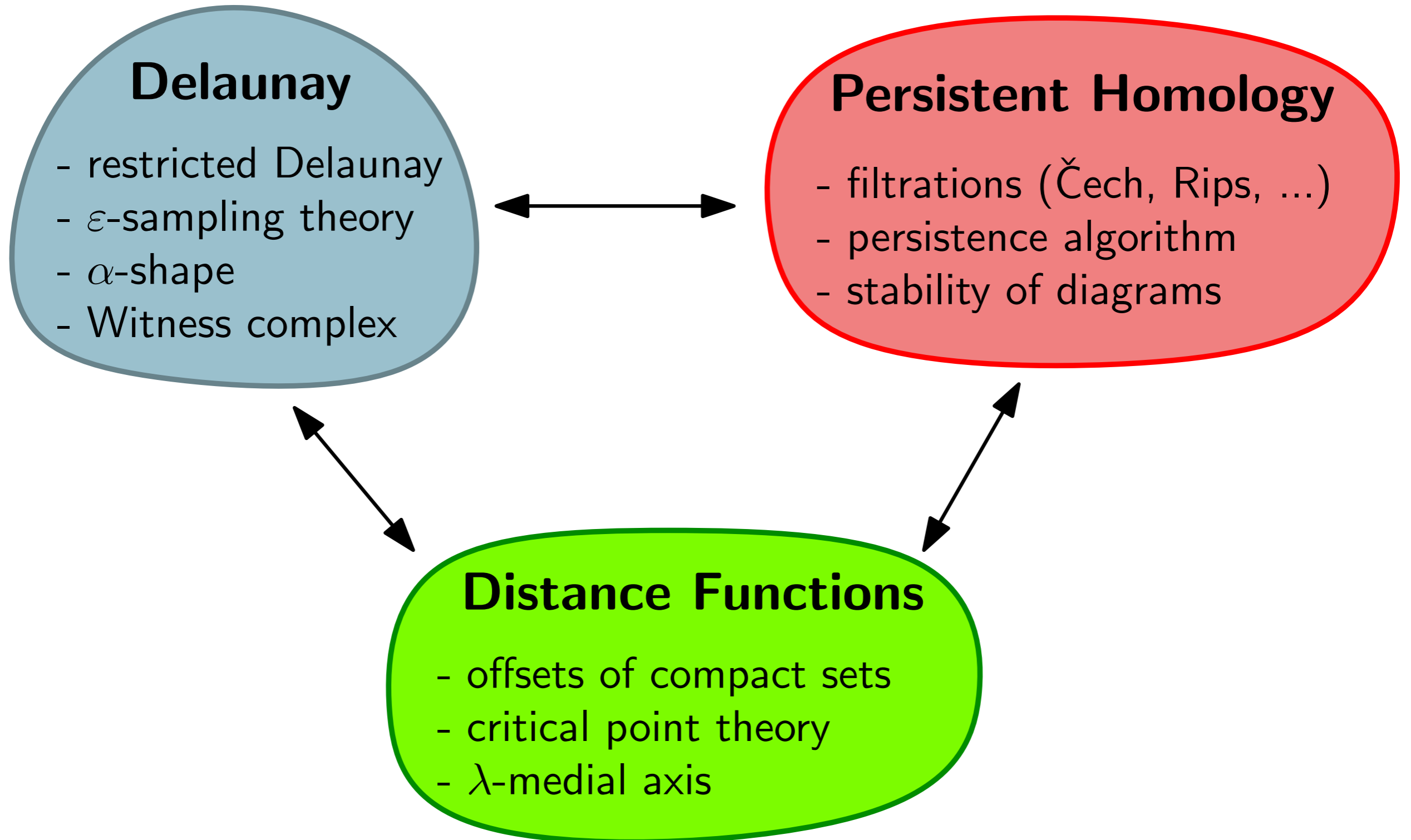
Persistent Homology

- filtrations (Čech, Rips, ...)
- persistence algorithm
- stability of diagrams

Distance Functions

- offsets of compact sets
- critical point theory
- λ -medial axis

Theoretical Tools



Some Achievements and Perspectives

Wednesday, July 8th		Thursday, July 9th	
08:30 - 09:00	Registration	09:00 - 09:20	Registration
09:00 - 09:20	Opening remarks	09:20 - 10:30	Key Note: An introduction to zigzag persistence Vin de Silva
09:20 - 10:30	Key Note: Geometric entropy minimization Alfred Hero	10:30 - 10:50	Coffee break
10:30 - 10:50	Coffee break	10:50 - 12:30	Session 4: Persistence and unsupervised learning
10:50 - 12:30	Session 1: Reconstruction in 3D		Persistence-based clustering Primož Skraba
	Scale space meshing Julie Digne		Persistent cohomology and circular coordinates Mikael Vejdemo-Johansson
	Reconstructing 3D compact sets Frédéric Cazals	12:30 - 14:00	Lunch
12:30 - 14:00	Lunch	14:00 - 15:40	Session 5: Signatures for shape classification
14:00 - 15:40	Session 2: Reconstruction in arbitrary dimensions		Topo-geometric Modeling for 3D objects Hamid Krim
	Manifold Reconstruction from Tangential Complex Arijit Ghosh		Gromov-Wasserstein stable signatures for object matching and the role of persistence Facundo Mémoli
	Model selection for simplicial approximation Bertrand Michel	15:40 - 16:00	Coffee break
15:40 - 16:00	Coffee break	16:00 - 17:40	Session 6: Shape matching
16:00 - 17:40	Session 3: Geometric inference in the presence of outliers		Heat Kernel Signature: A Concise and Provably Informative Multi-scale Signature Based on Heat Diffusion Maksims Ovsjanikovs
	Geometric Inference for Measures based on Distance Functions Quentin Mérigot		Deformable shape matching using linear programming Qixing Huang
	Efficient Approximation of the Distance to an Empirical Measure Dmitry Morozov		
Friday, July 10th			
09:00 - 10:40	Session 7: Reconstruction and mesh generation in 3D		
	Finite Element Analysis of Computer Aided Design Assembly Kirill Pichon Gostaf		
	Reconstruction from Cross-Sections Pooran Memari		
10:40 - 11:00	Coffee break		
11:00 - 12:40	Session 8: Delaunay triangulations		
	Periodic Delaunay triangulations Manuel Caroli		
	A compact data structure to represent the Delaunay Triangulation Clément Maria		
12:40 - 13:00	Closing remarks		

Some Achievements and Perspectives

Wednesday, July 8th

08:30 - 09:00	Registration
09:00 - 09:20	Opening remarks
09:20 - 10:30	Key Note: Geometric entropy minimization Alfred Hero
10:30 - 10:50	Coffee break
10:50 - 12:30	Session 1: Reconstruction in 3D Scale space meshing Julie Digne Reconstructing 3D compact sets Frédéric Cazals
12:30 - 14:00	Lunch
14:00 - 15:40	Session 2: Reconstruction in arbitrary dimensions Manifold Reconstruction from Tangential Complex Arijit Ghosh Model selection for simplicial approximation Bertrand Michel
15:40 - 16:00	Coffee break
16:00 - 17:40	Session 3: Geometric inference in the presence of outliers Geometric Inference for Measures based on Distance Functions Quentin Mérigot Efficient Approximation of the Distance to an Empirical Measure Dmitry Morozov

Thursday, July 9th

09:00 - 09:20	Registration
09:20 - 10:30	Key Note: An introduction to zigzag persistence Vin de Silva
10:30 - 10:50	Coffee break
10:50 - 12:30	Session 4: Persistence and unsupervised learning Persistence-based clustering Primož Skraba Persistent cohomology and circular coordinates Mikael Vejdemo-Johansson
12:30 - 14:00	Lunch
14:00 - 15:40	Session 5: Signatures for shape classification Topo-geometric Modeling for 3D objects Hamid Krim Gromov-Wasserstein stable signatures for object matching and the role of persistence Facundo Mémoli
15:40 - 16:00	Coffee break
16:00 - 17:40	Session 6: Shape matching Heat Kernel Signature: A Concise and Provably Informative Multi-scale Signature Based on Heat Diffusion Maksims Ovsjanikovs Deformable shape matching using linear programming Qixing Huang

Friday, July 10th

09:00 - 10:40	Session 7: Reconstruction and mesh generation in 3D Finite Element Analysis of Computer Aided Design Assembly Kirill Pichon Gostaf Reconstruction from Cross-Sections Pooran Memari
10:40 - 11:00	Coffee break
11:00 - 12:40	Session 8: Delaunay triangulations Periodic Delaunay triangulations Manuel Caroli A compact data structure to represent the Delaunay Triangulation Clément Maria
12:40 - 13:00	Closing remarks

Some Achievements and Perspectives

Wednesday, July 8th

08:30 - 09:00	Registration
09:00 - 09:20	Opening remarks
09:20 - 10:30	Key Note: Geometric entropy minimization Alfred Hero
10:30 - 10:50	Coffee break
10:50 - 12:30	Session 1: Reconstruction in 3D Scale space meshing Julie Digne Reconstructing 3D compact sets Frédéric Cazals
12:30 - 14:00	Lunch
14:00 - 15:40	Session 2: Reconstruction in arbitrary dimensions Manifold Reconstruction from Tangential Complex Arijit Ghosh Model selection for simplicial approximation Bertrand Michel
15:40 - 16:00	Coffee break
16:00 - 17:40	Session 3: Geometric inference in the presence of outliers Geometric Inference for Measures based on Distance Functions Quentin Mérigot Efficient Approximation of the Distance to an Empirical Measure Dmitry Morozov

Thursday, July 9th

09:00 - 09:20	Registration
09:20 - 10:30	Key Note: An introduction to zigzag persistence Vin de Silva
10:30 - 10:50	Coffee break
10:50 - 12:30	Session 4: Persistence and unsupervised learning Persistence-based clustering Primož Skraba Persistent cohomology and circular coordinates Mikael Vejdemo-Johansson
12:30 - 14:00	Lunch
14:00 - 15:40	Session 5: Signatures for shape classification Topo-geometric Modeling for 3D objects Hamid Krim Gromov-Wasserstein stable signatures for object matching and the role of persistence Facundo Mémoli
15:40 - 16:00	Coffee break
16:00 - 17:40	Session 6: Shape matching Heat Kernel Signature: A Concise and Provably Informative Multi-scale Signature Based on Heat Diffusion Maksims Ovsjanikovs Deformable shape matching using linear programming Qixing Huang

Friday, July 10th

09:00 - 10:40	Session 7: Reconstruction and mesh generation in 3D Finite Element Analysis of Computer Aided Design Assembly Kirill Pichon Gostaf Reconstruction from Cross-Sections Pooran Memari
10:40 - 11:00	Coffee break
11:00 - 12:40	Session 8: Delaunay triangulations Periodic Delaunay triangulations Manuel Caroli A compact data structure to represent the Delaunay Triangulation Clément Maria
12:40 - 13:00	Closing remarks

Some Achievements and Perspectives

Wednesday, July 8th

08:30 - 09:00	Registration
09:00 - 09:20	Opening remarks
09:20 - 10:30	Key Note: Geometric entropy minimization Alfred Hero
10:30 - 10:50	Coffee break
10:50 - 12:30	Session 1: Reconstruction in 3D Scale space meshing Julie Digne
	Reconstructing 3D compact sets Frédéric Cazals
12:30 - 14:00	Lunch
14:00 - 15:40	Session 2: Reconstruction in arbitrary dimensions Manifold Reconstruction from Tangential Complex Arijit Ghosh
	Model selection for simplicial approximation Bertrand Michel
15:40 - 16:00	Coffee break
16:00 - 17:40	Session 3: Geometric inference in the presence of outliers Geometric Inference for Measures based on Distance Functions Quentin Mérigot
	Efficient Approximation of the Distance to an Empirical Measure Dmitry Morozov

Thursday, July 9th

09:00 - 09:20	Registration
09:20 - 10:30	Key Note: An introduction to zigzag persistence Vin de Silva
10:30 - 10:50	Coffee break
10:50 - 12:30	Session 4: Persistence and unsupervised learning Persistence-based clustering Primož Skraba
	Persistent cohomology and circular coordinates Mikael Vejdemo-Johansson
12:30 - 14:00	Lunch
14:00 - 15:40	Session 5: Signatures for shape classification Topo-geometric Modeling for 3D objects Hamid Krim
	Gromov-Wasserstein stable signatures for object matching and the role of persistence Facundo Mémoli
15:40 - 16:00	Coffee break
16:00 - 17:40	Session 6: Shape matching Heat Kernel Signature: A Concise and Provably Informative Multi-scale Signature Based on Heat Diffusion Maksims Ovsjanikovs
	Deformable shape matching using linear programming Qixing Huang

Friday, July 10th

09:00 - 10:40	Session 7: Reconstruction and mesh generation in 3D Finite Element Analysis of Computer Aided Design Assembly Kirill Pichon Gostaf
	Reconstruction from Cross-Sections Pooran Memari
10:40 - 11:00	Coffee break
11:00 - 12:40	Session 8: Delaunay triangulations Periodic Delaunay triangulations Manuel Caroli
	A compact data structure to represent the Delaunay Triangulation Clément Maria
12:40 - 13:00	Closing remarks