Technische Universität München
Mathematical Methods for High-Dimensional Data Analysis
July 18 - 22, 2016

Topological Descriptors for Geometric Data

Steve Oudot

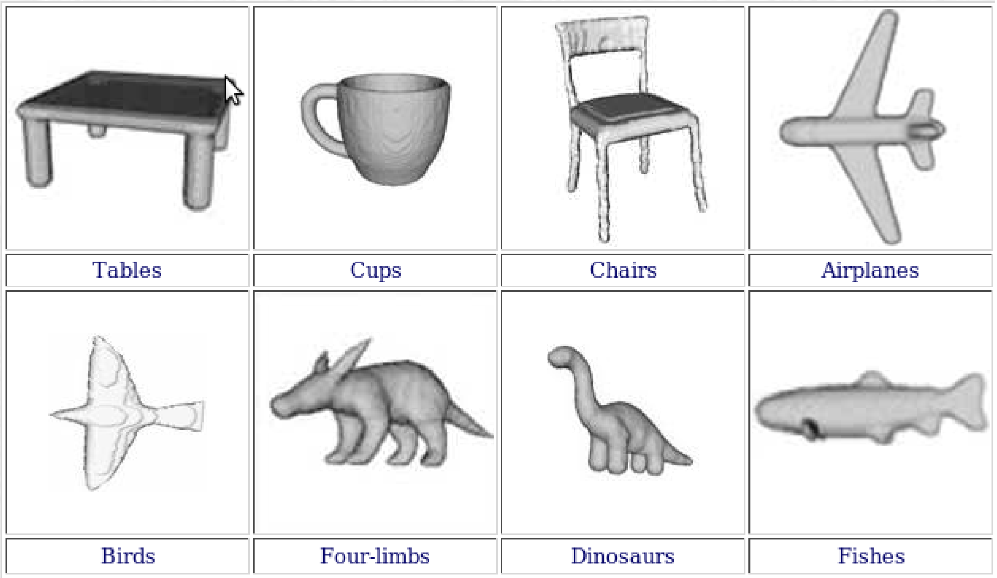
Resources:

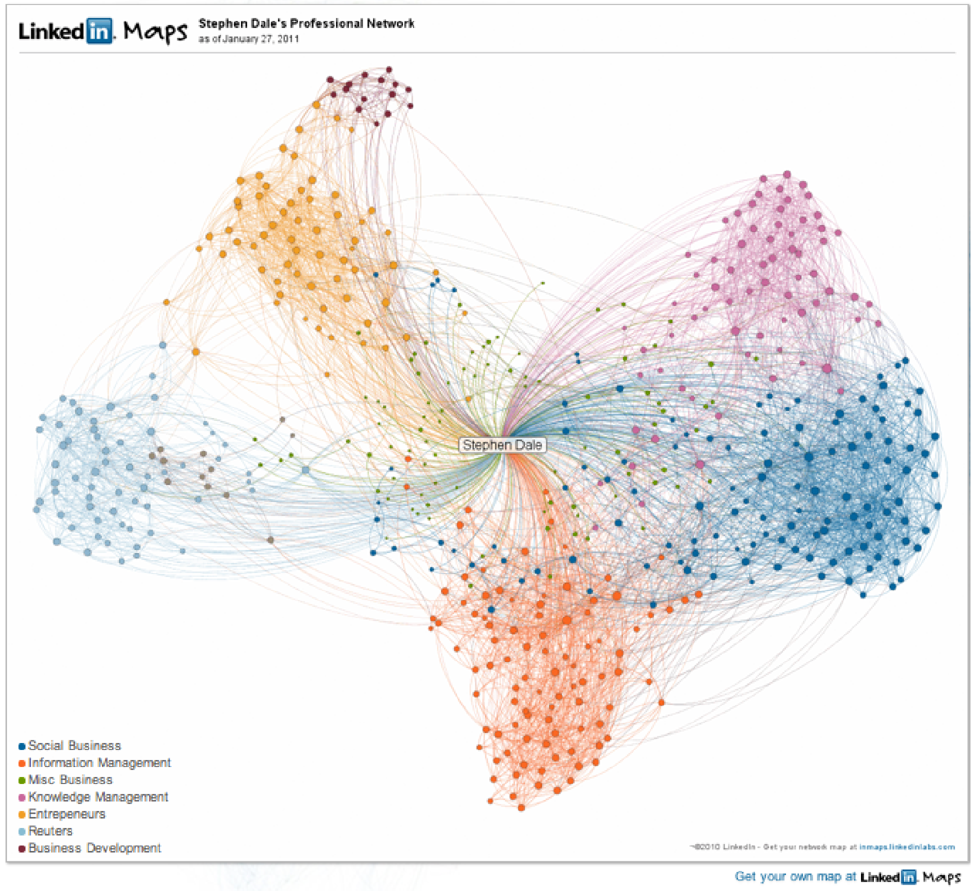
- `http://geometrica.saclay.inria.fr/team/Steve.Oudot/courses/TUM/`
- H. Edelsbrunner and J. Harer. *Computational topology: an introduction*. American Mathematical Society, 2010.
- S.O. *Persistence Theory: from Quiver Representations to Data Analysis*. AMS Mathematical Surveys and Monographs (209), 2015.

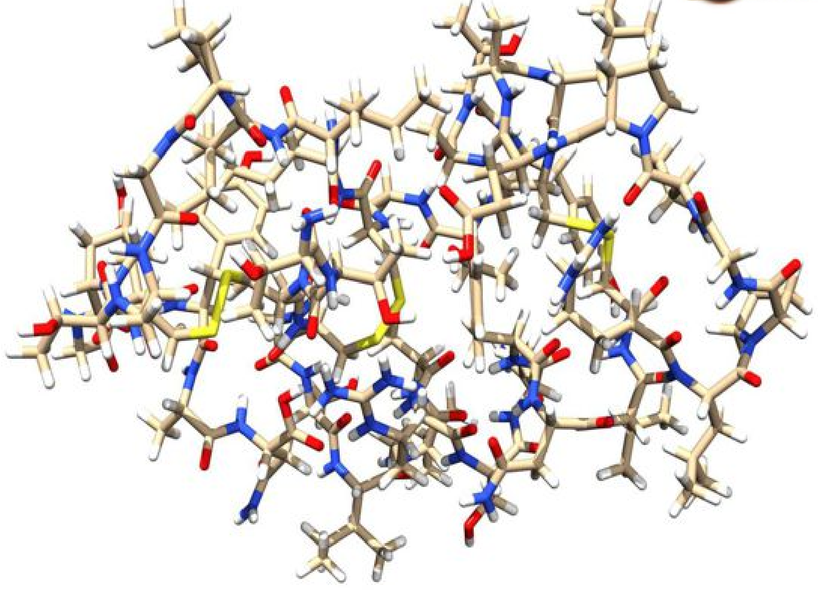
Geometric Data

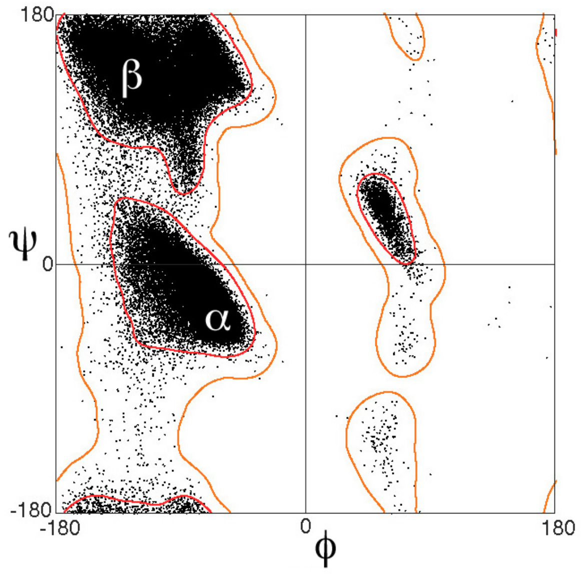
Input: point cloud equipped with a metric or (dis-)similarity measure

data point ≡ image/patch, geometric shape, protein conformation, patient, LinkedIn user...









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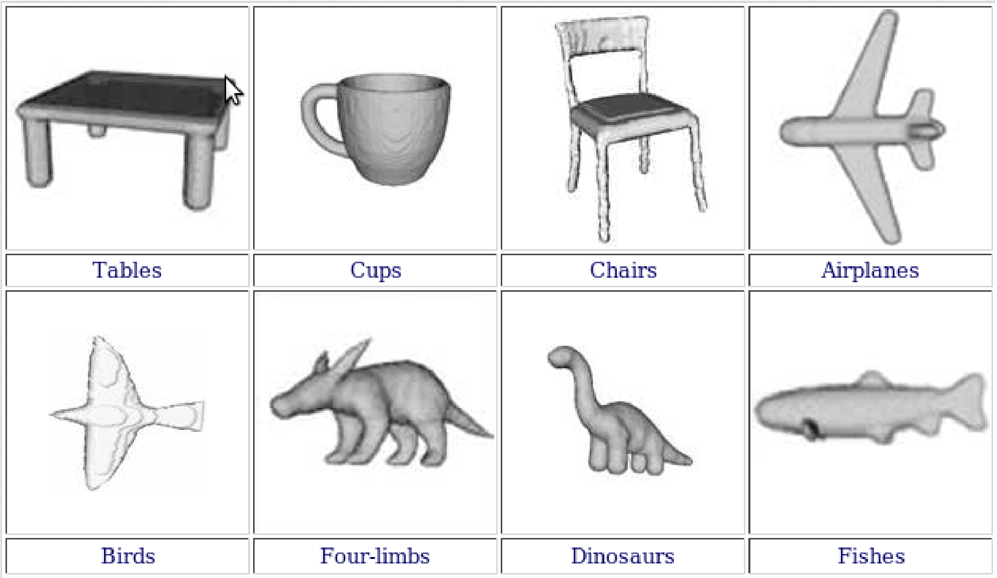
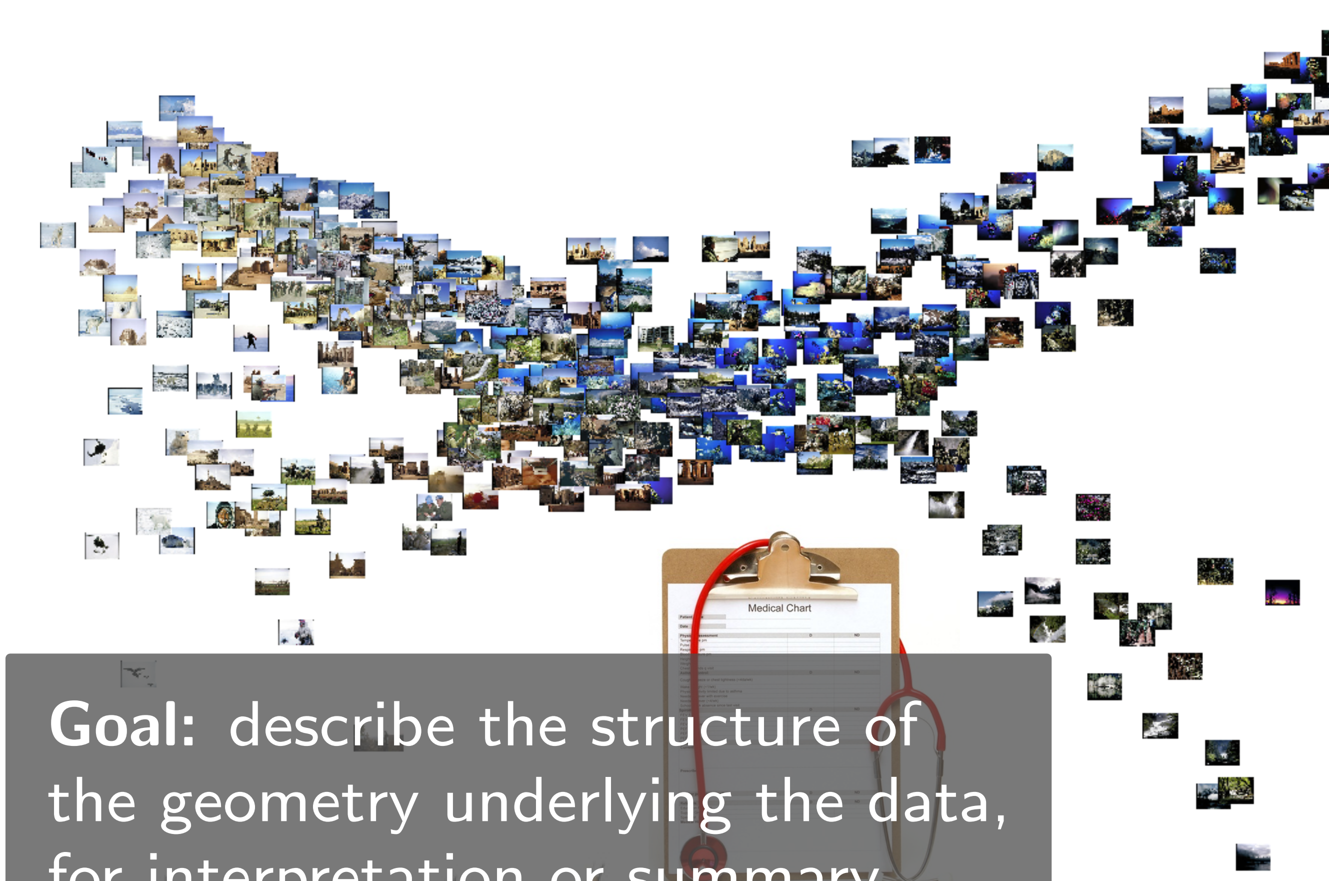
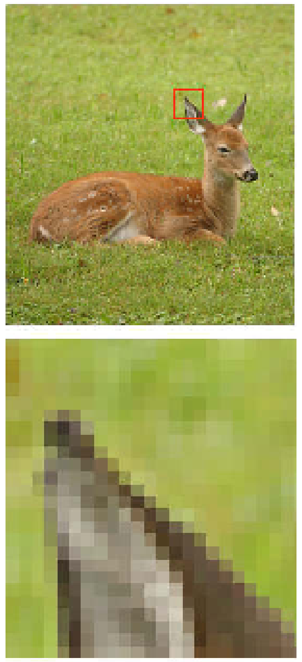
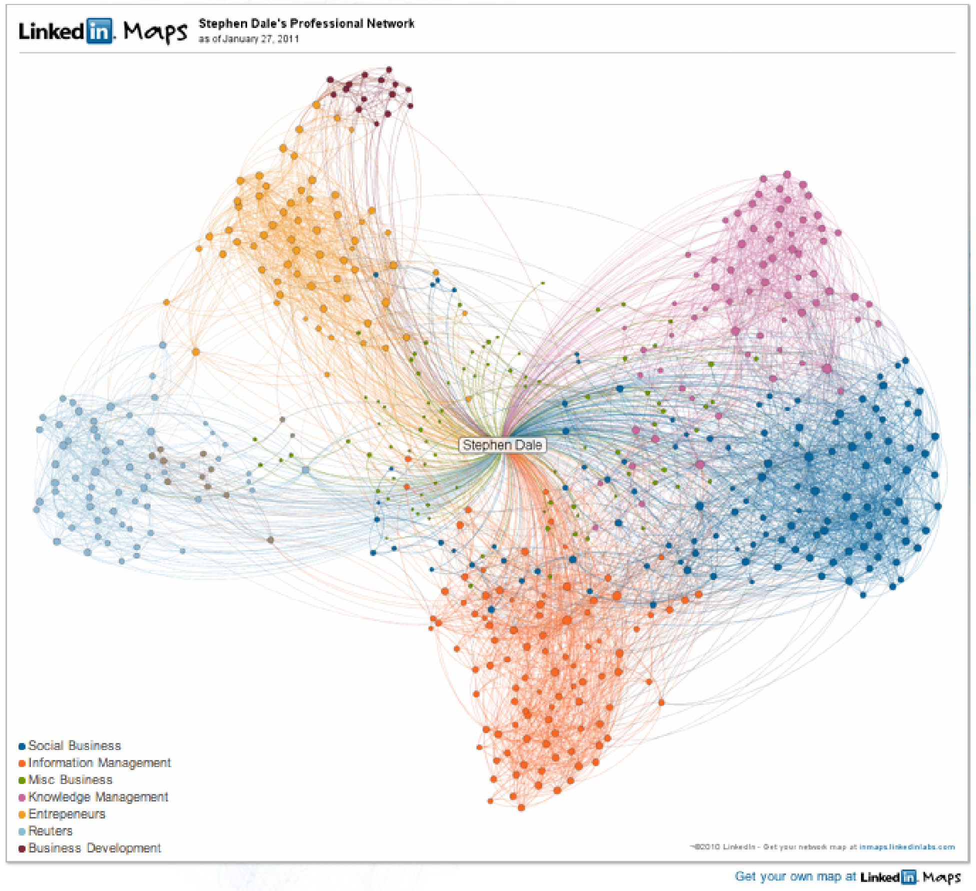
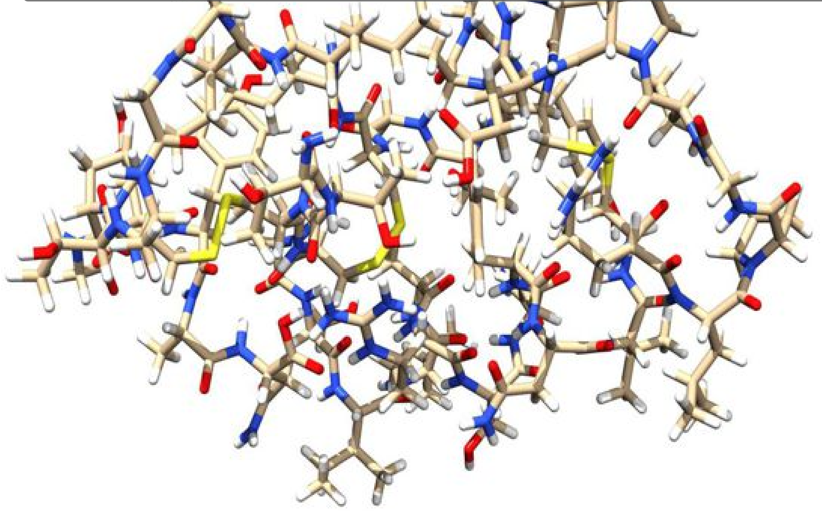


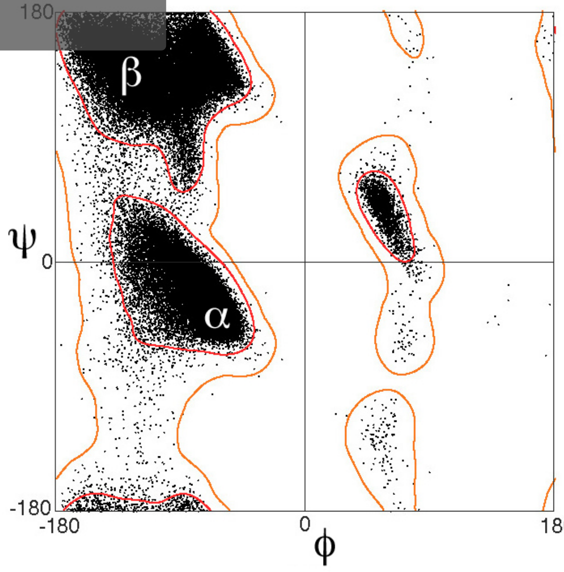




Goal: describe the structure of the geometry underlying the data, for interpretation or summary



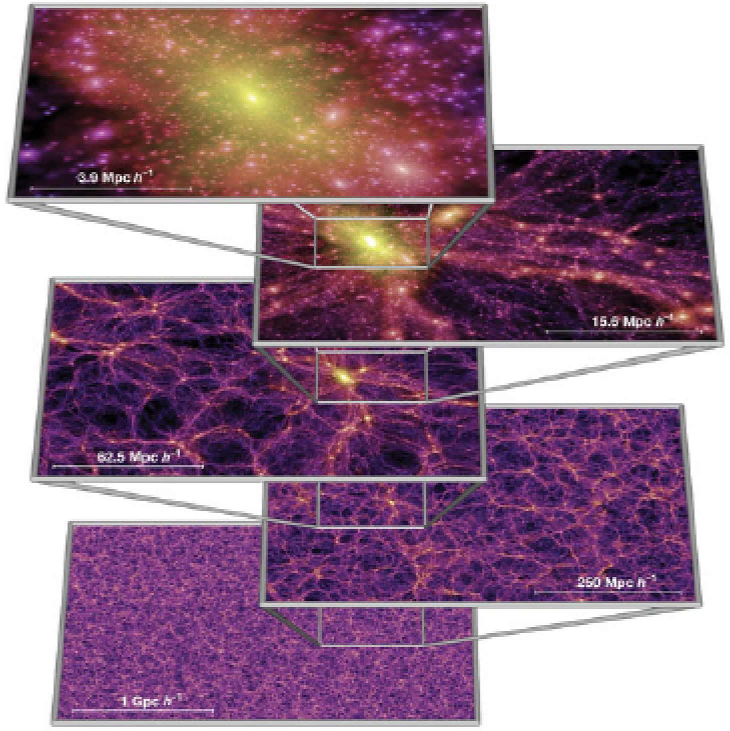


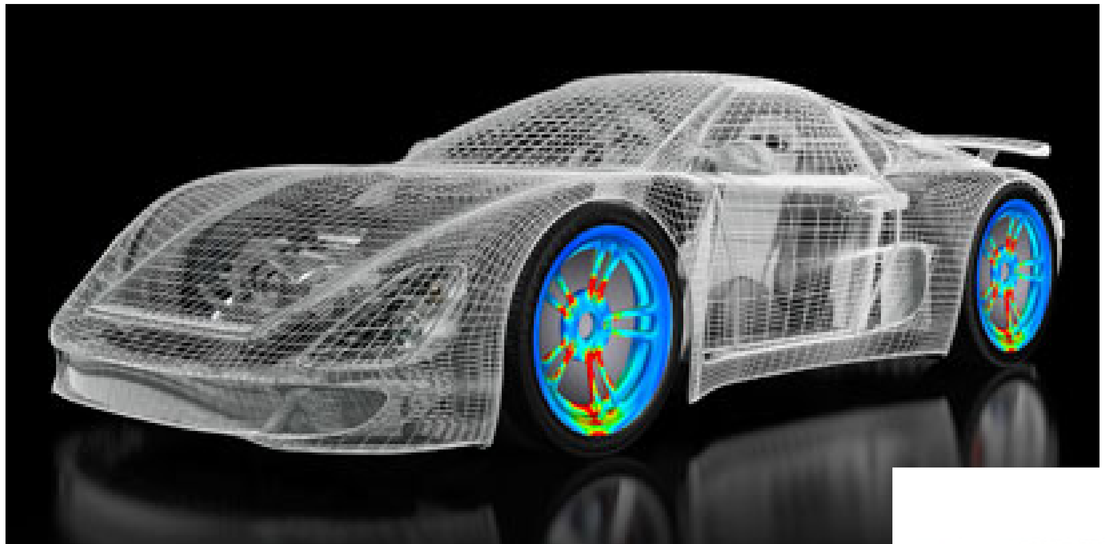


Context: the data deluge

Data are becoming more and more massive and complex:

- academia
- industry
- general public





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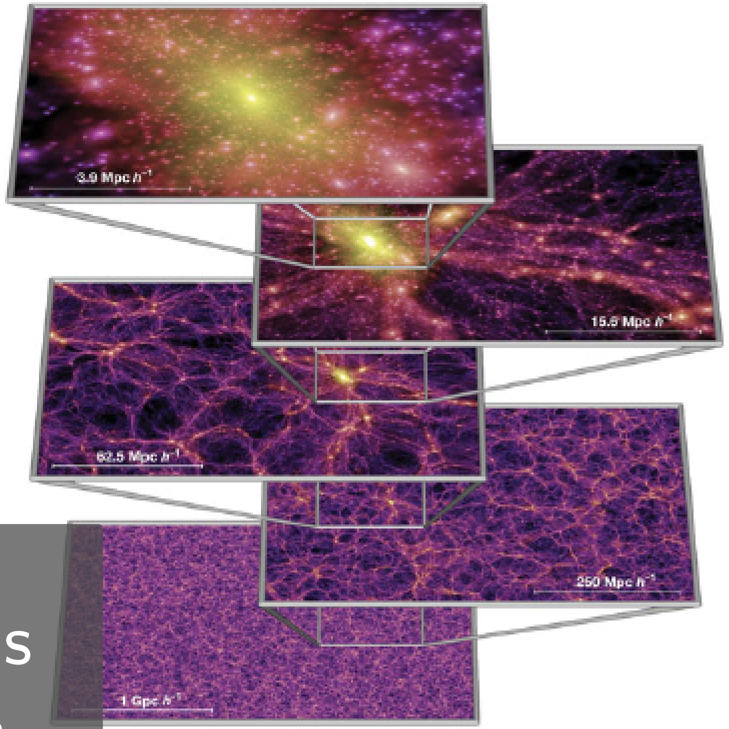
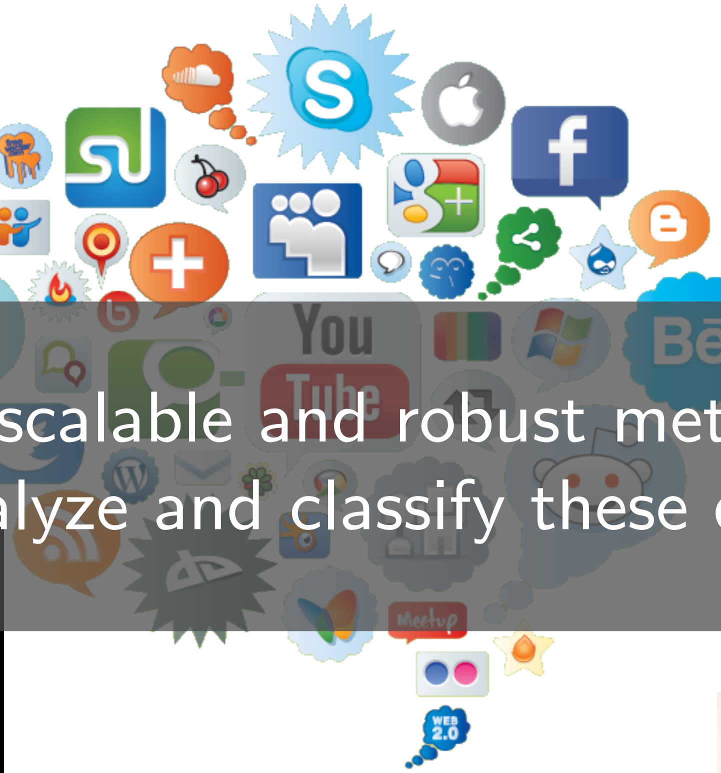
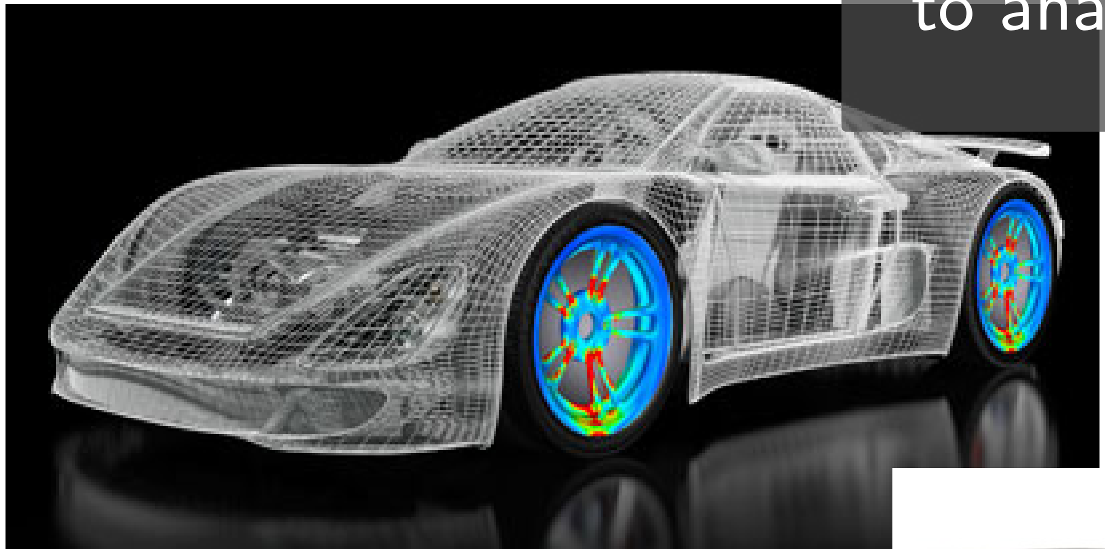
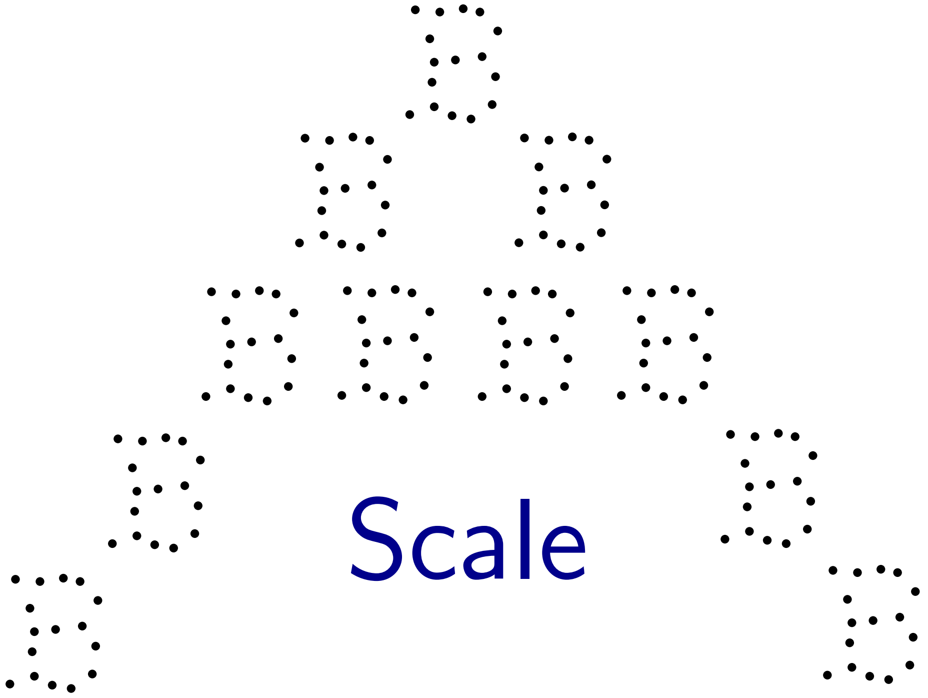




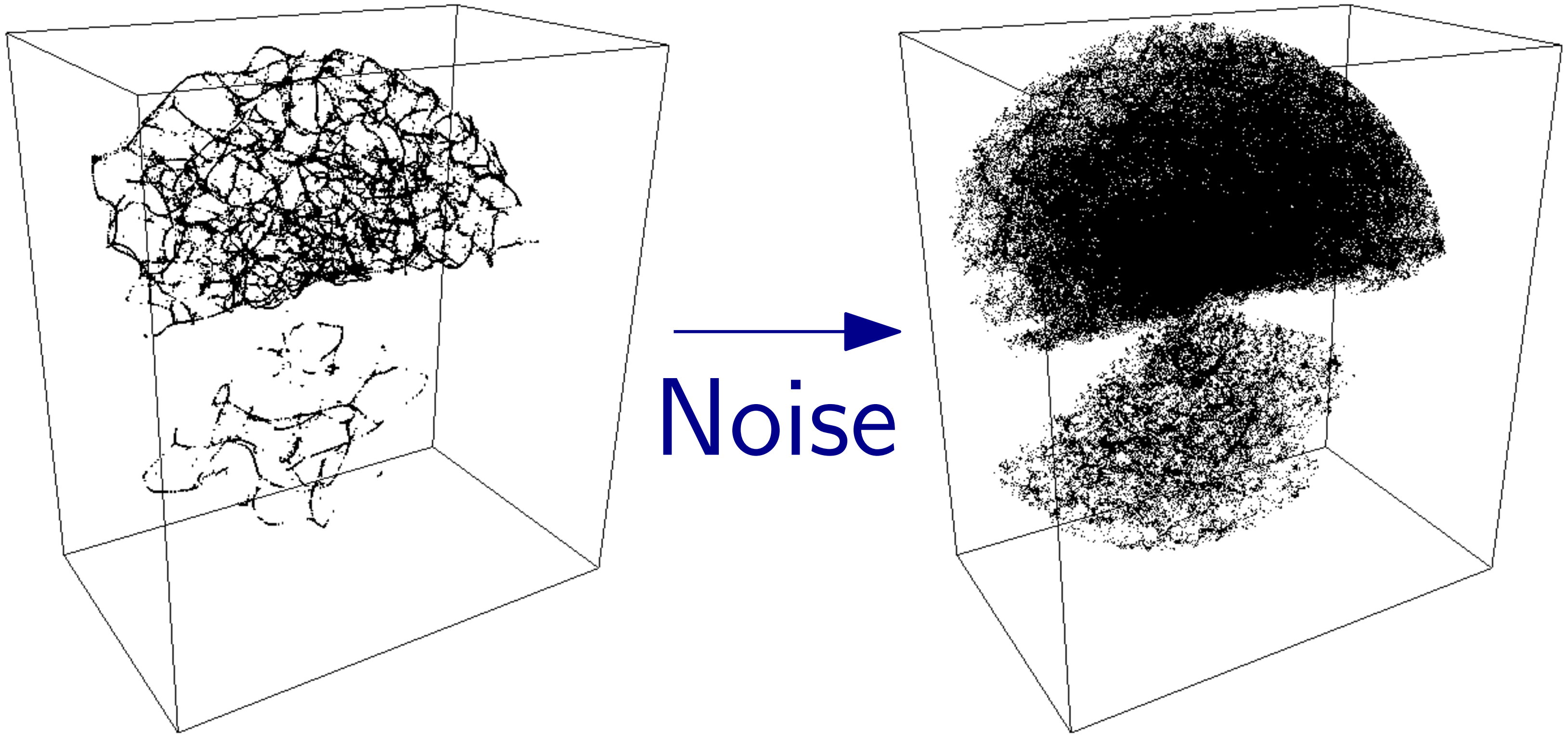
Need scalable and robust methods to analyze and classify these data



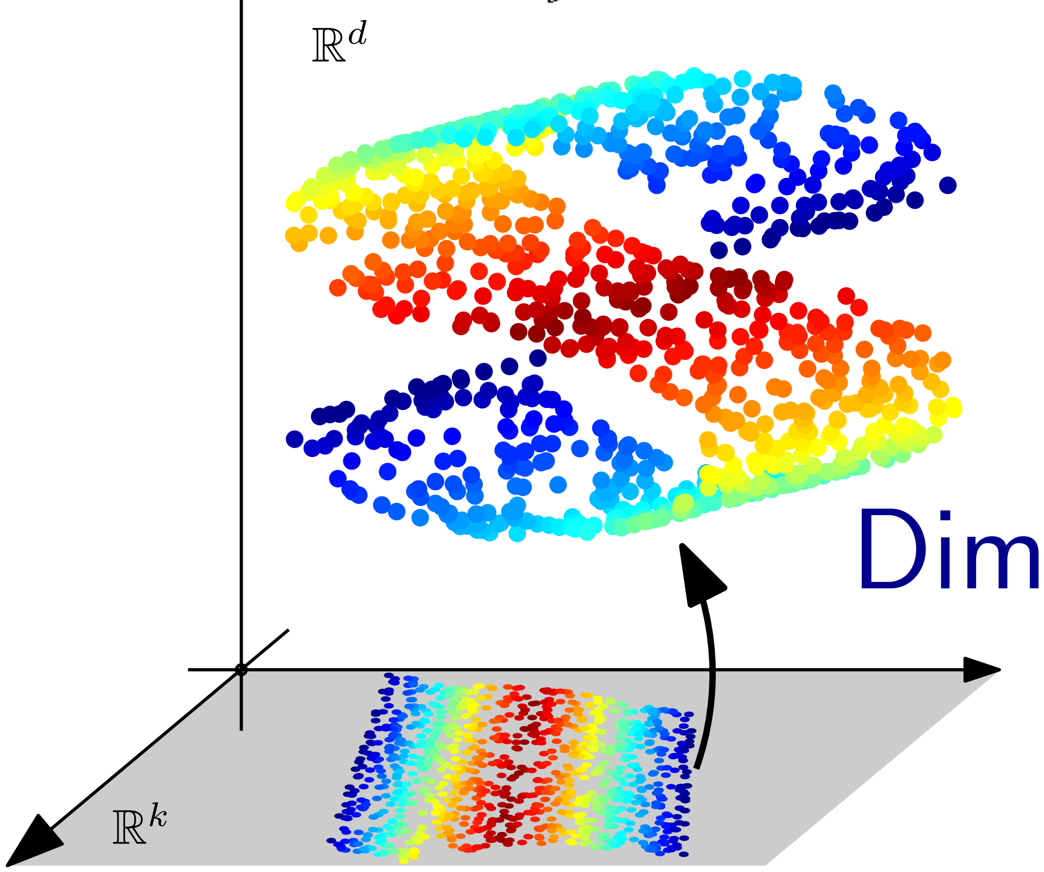
Challenges



Scale

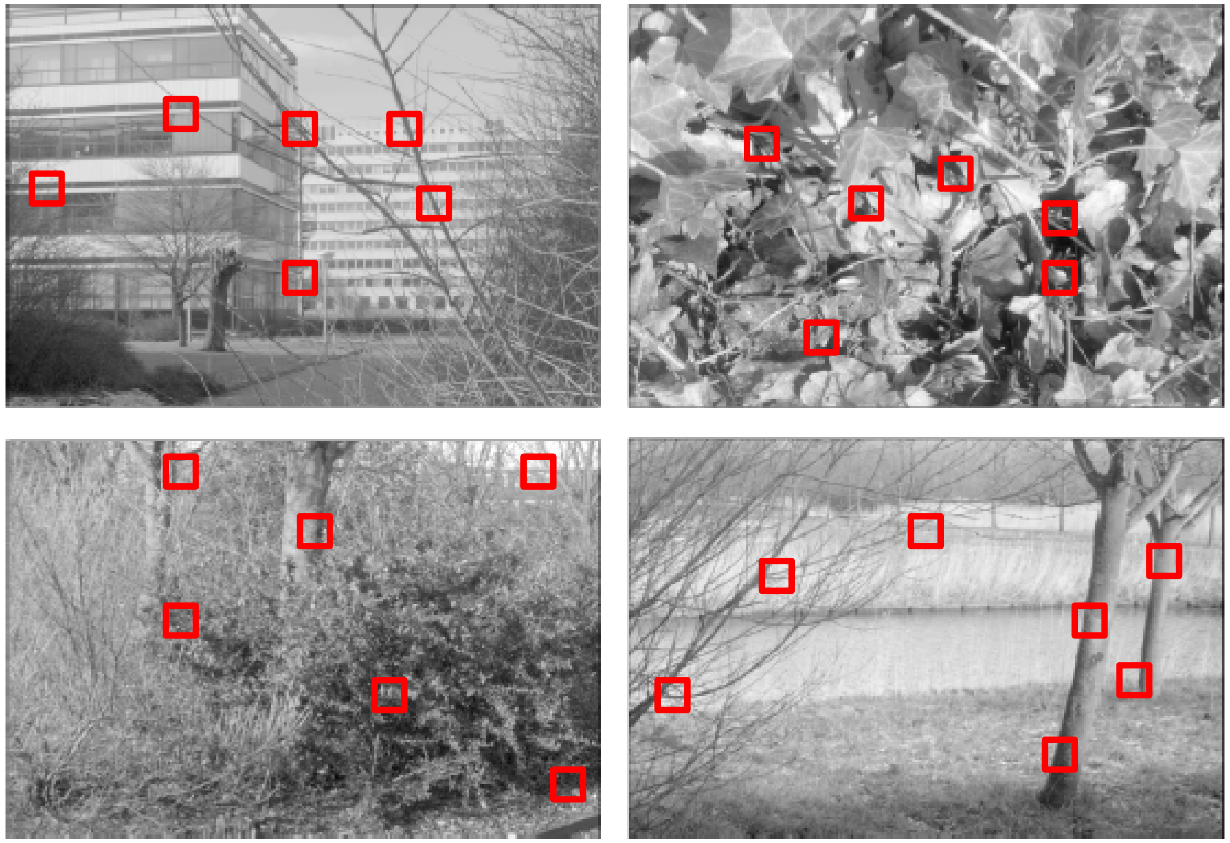


Noise



Dimensionality

Challenges

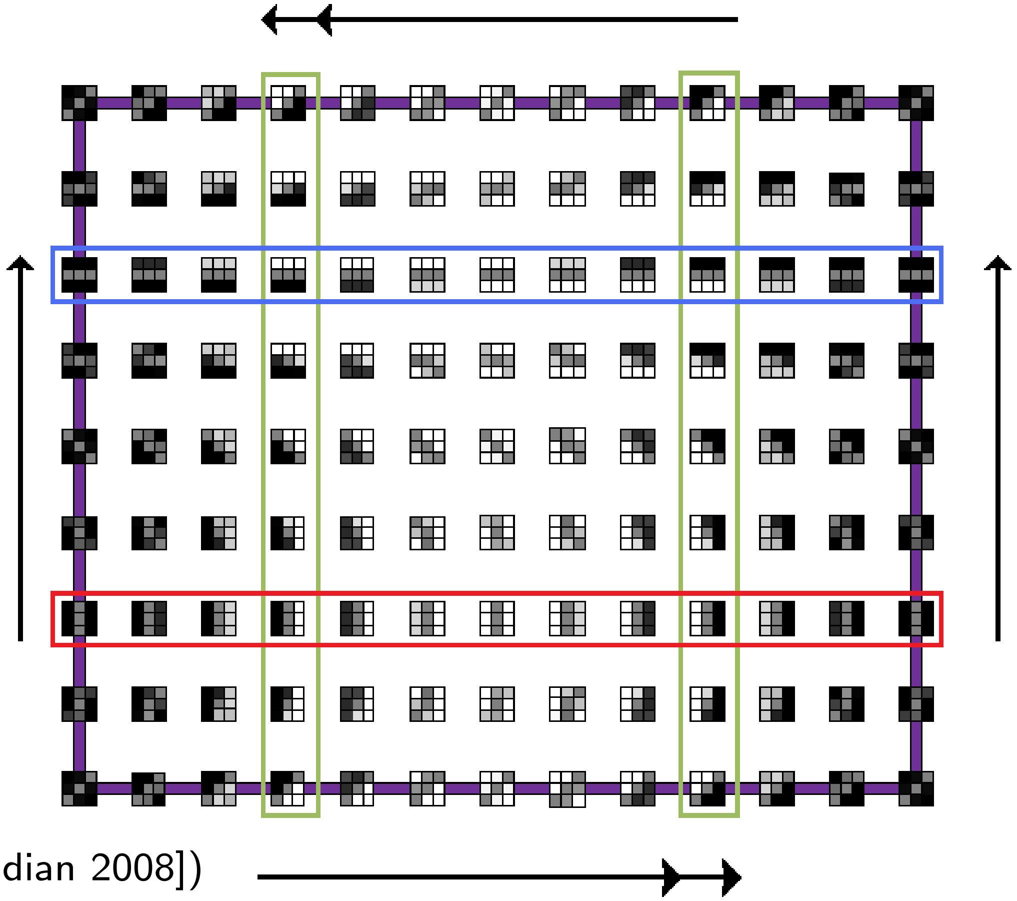


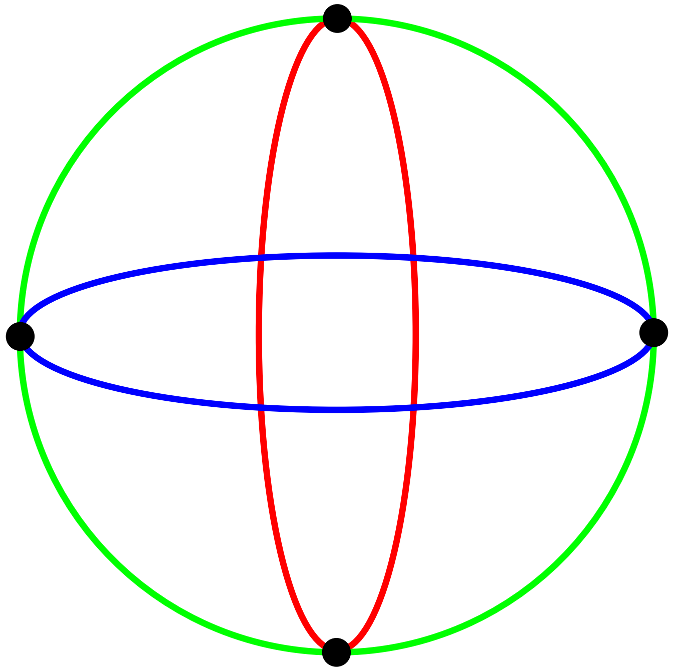
4 million data points in $\mathbb{R}^9$

(source: [Lee, Pederson, Mumford 2003])

Motivation: study cognitive representation of space of images

Topology





(source: [Carlsson, Ishkhanov, de Silva, Zomorodian 2008])

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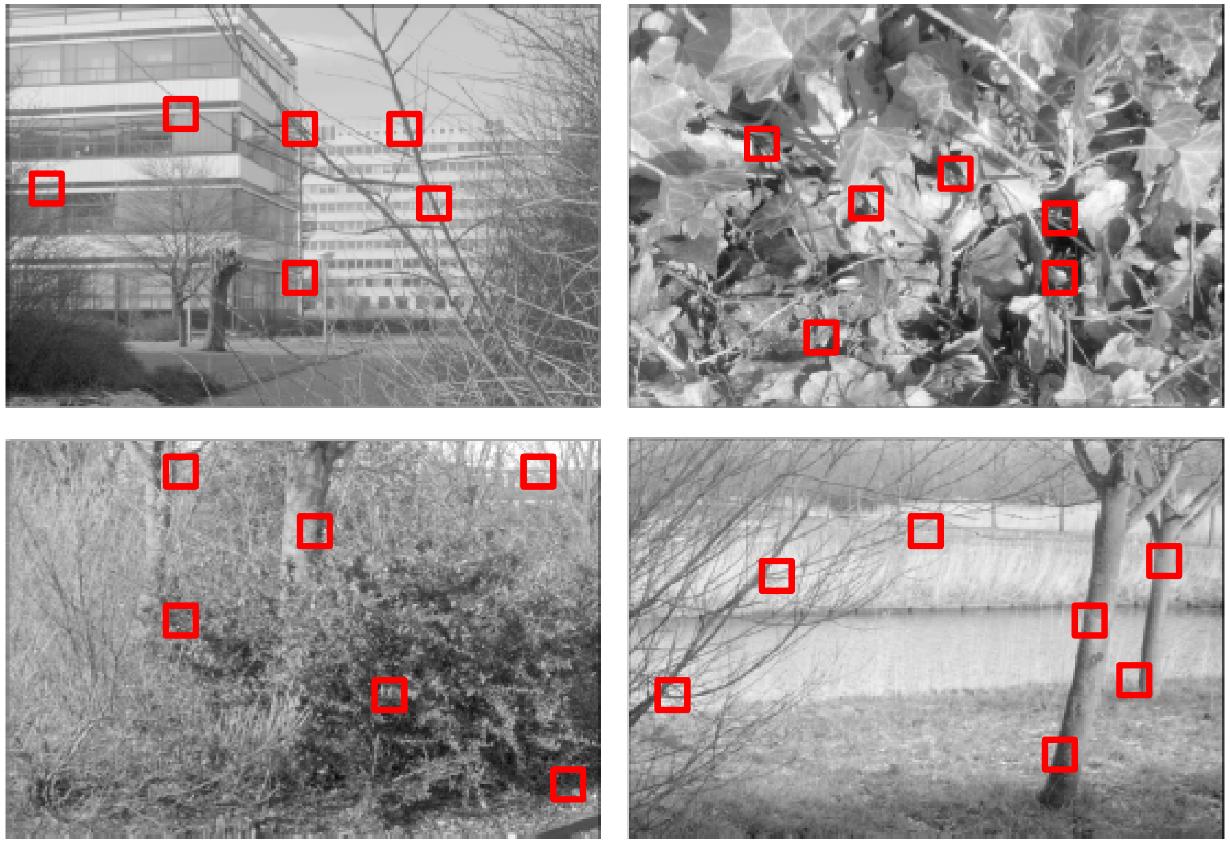

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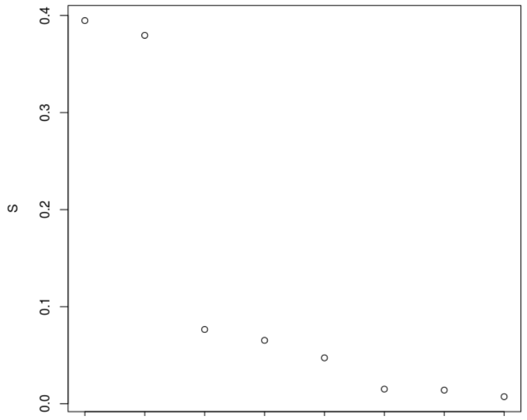

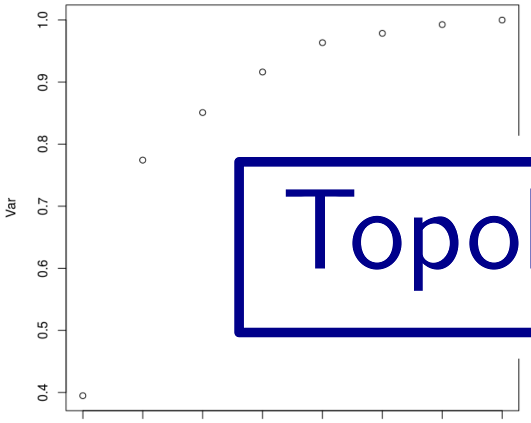

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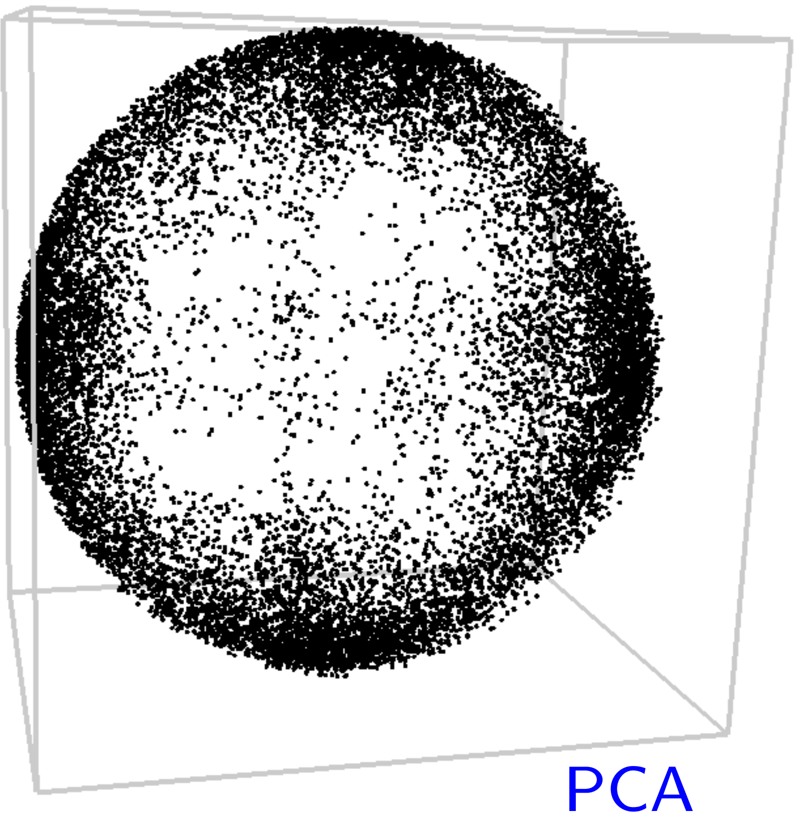
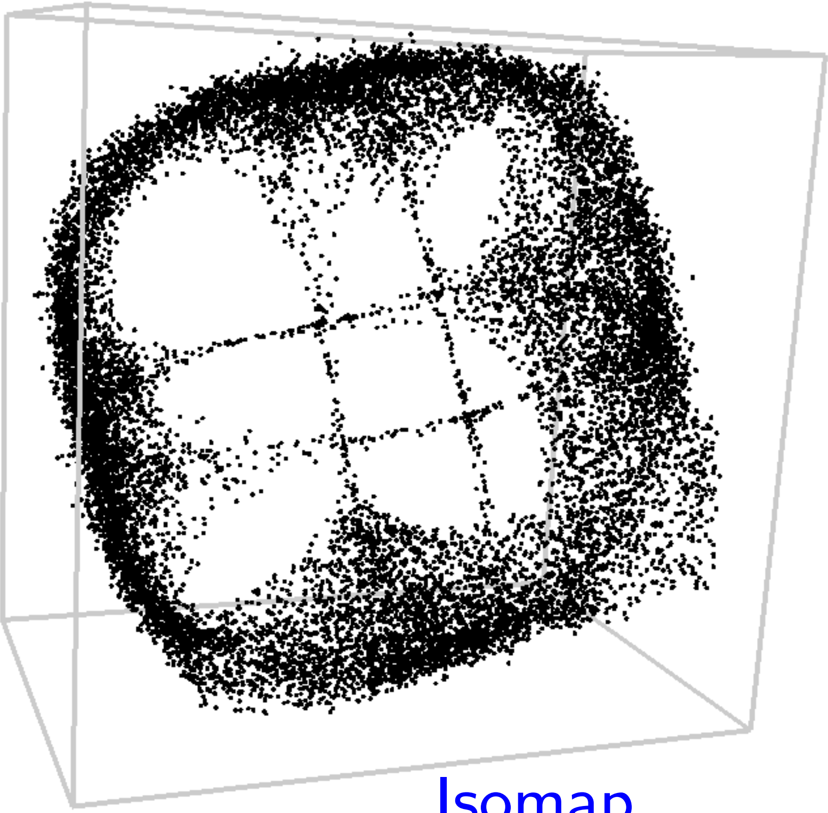
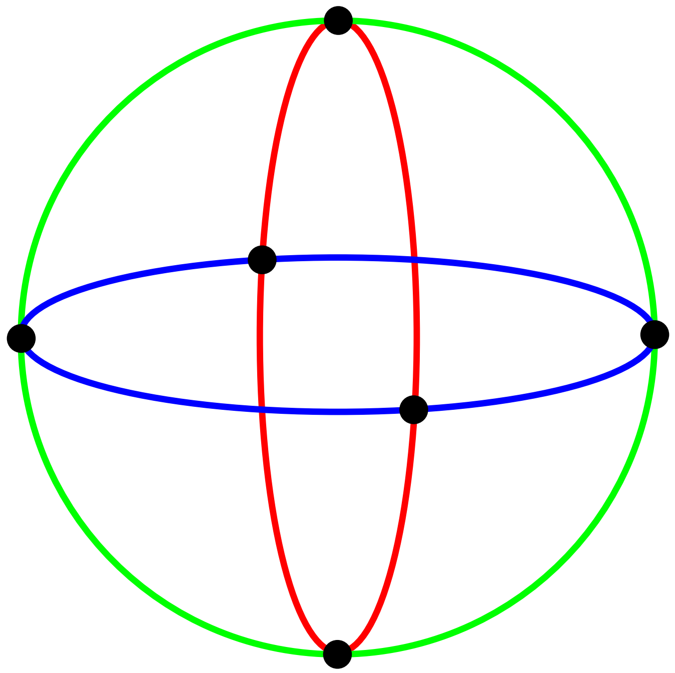


PCA



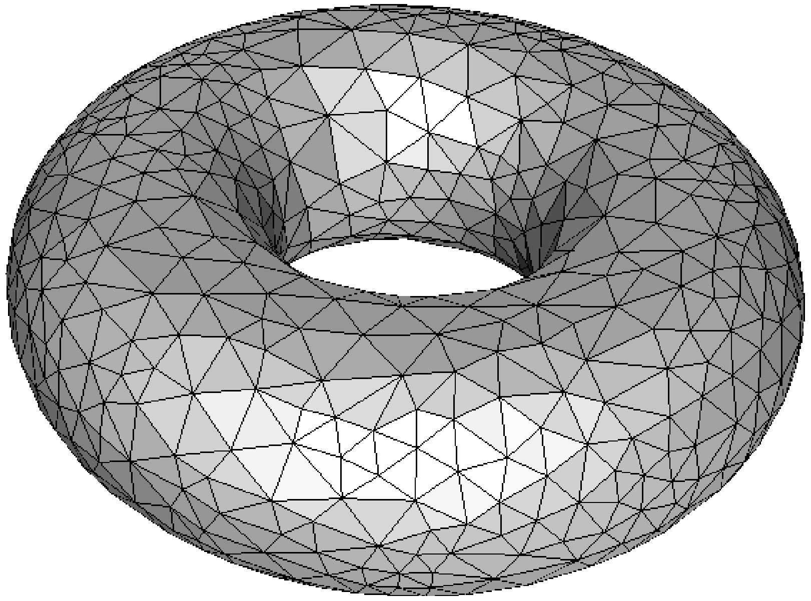
Isomap



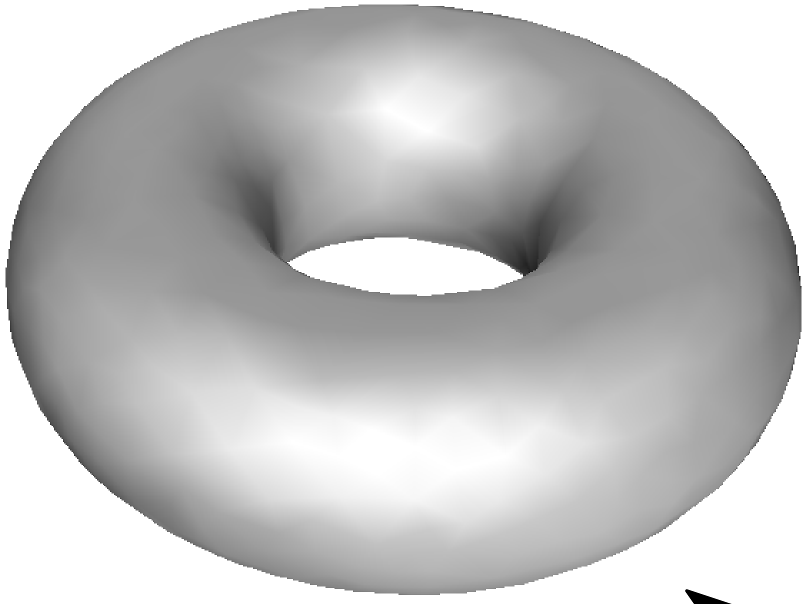
Topological Data Analysis (TDA)

topological invariants for classification

$$\beta_0 = \beta_2 = 1$$
$$\beta_1 = 2$$



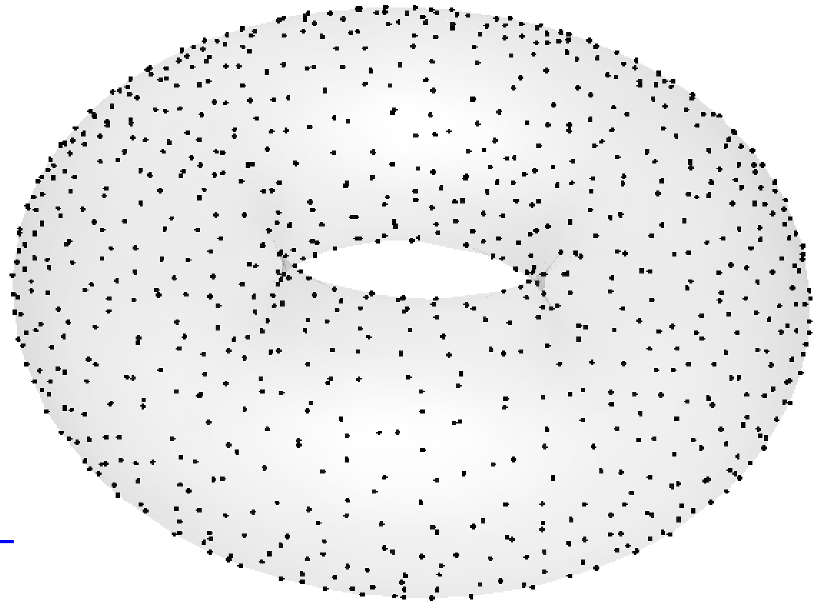
triangulation



compact set

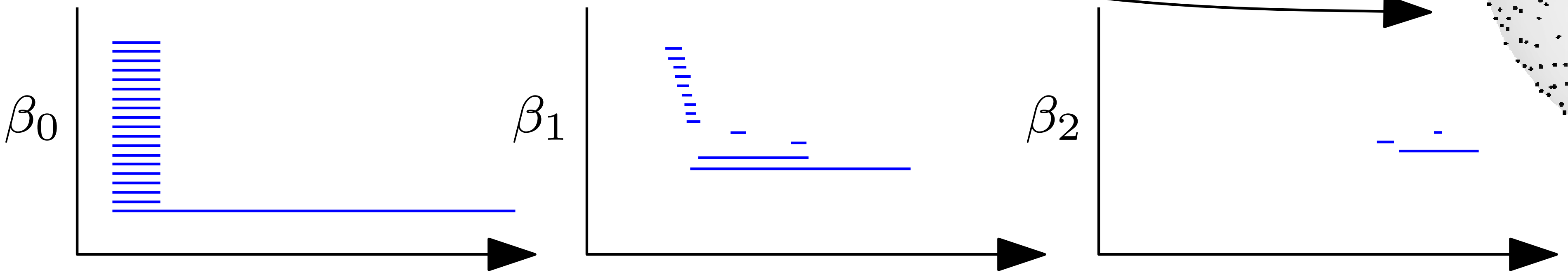
Algebraic topology in the 20th century

Algebraic topology in the 21st century



point cloud

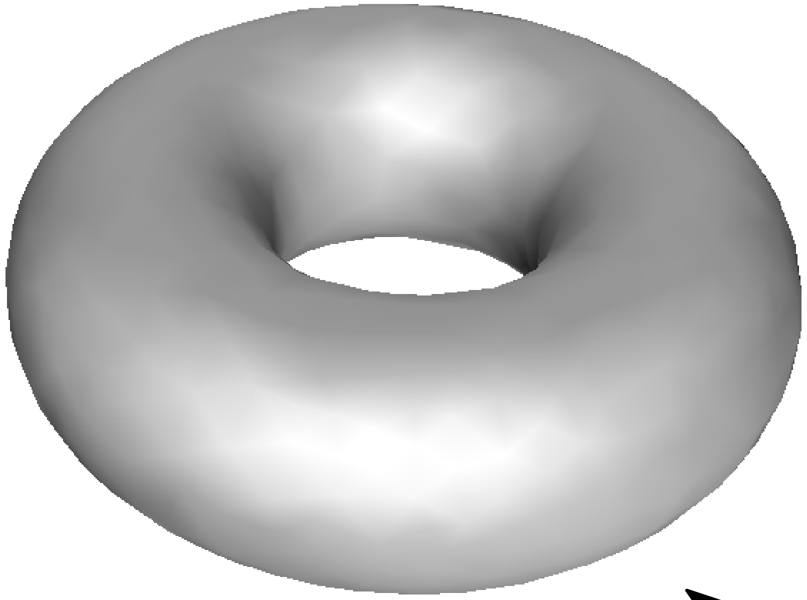
topological descriptors for inference and comparison



Topological Data Analysis (TDA)

Properties of topological descriptors:

- invariant under coordinate changes
- stable with respect to perturbations
- informative



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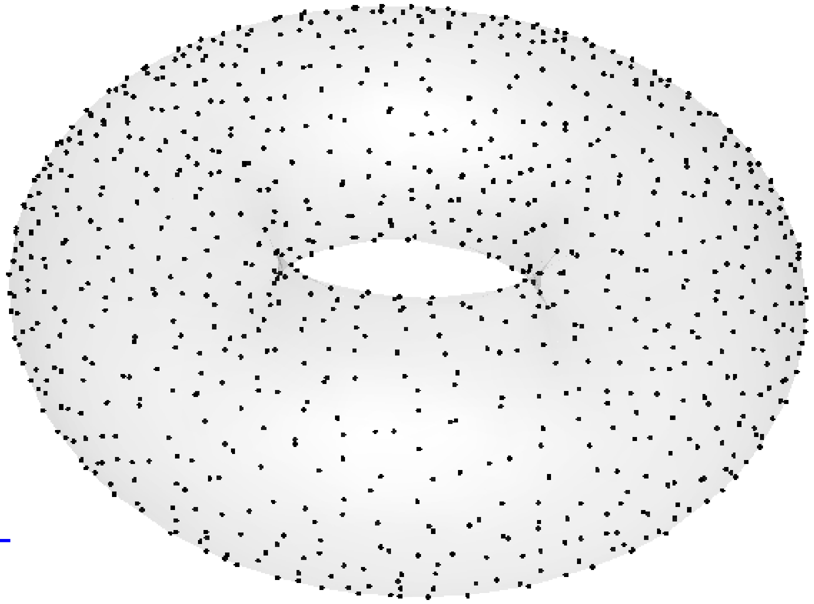

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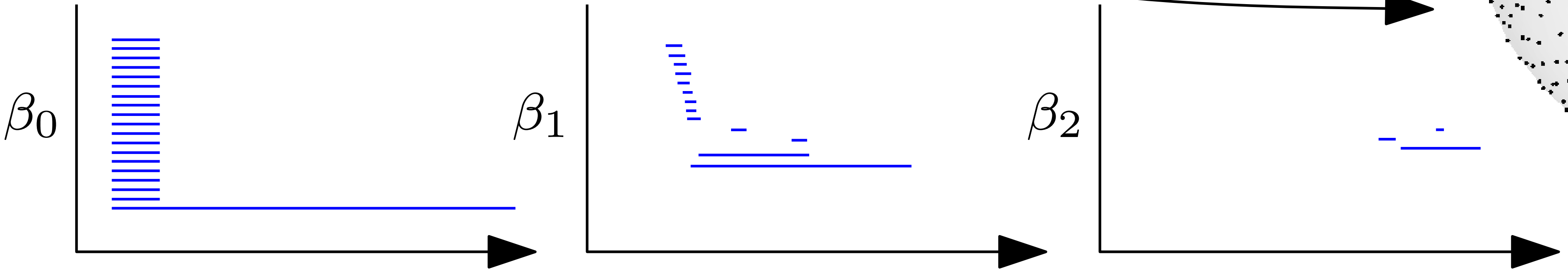
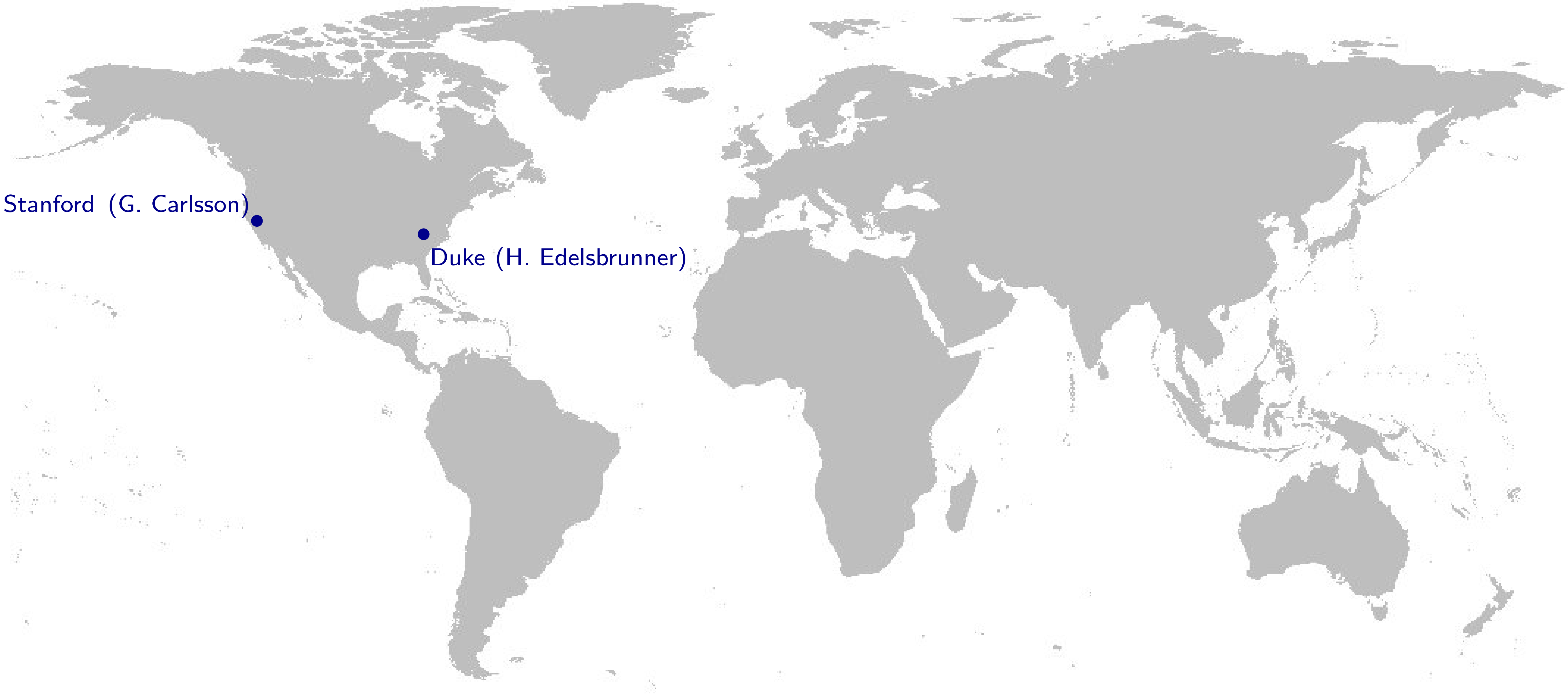
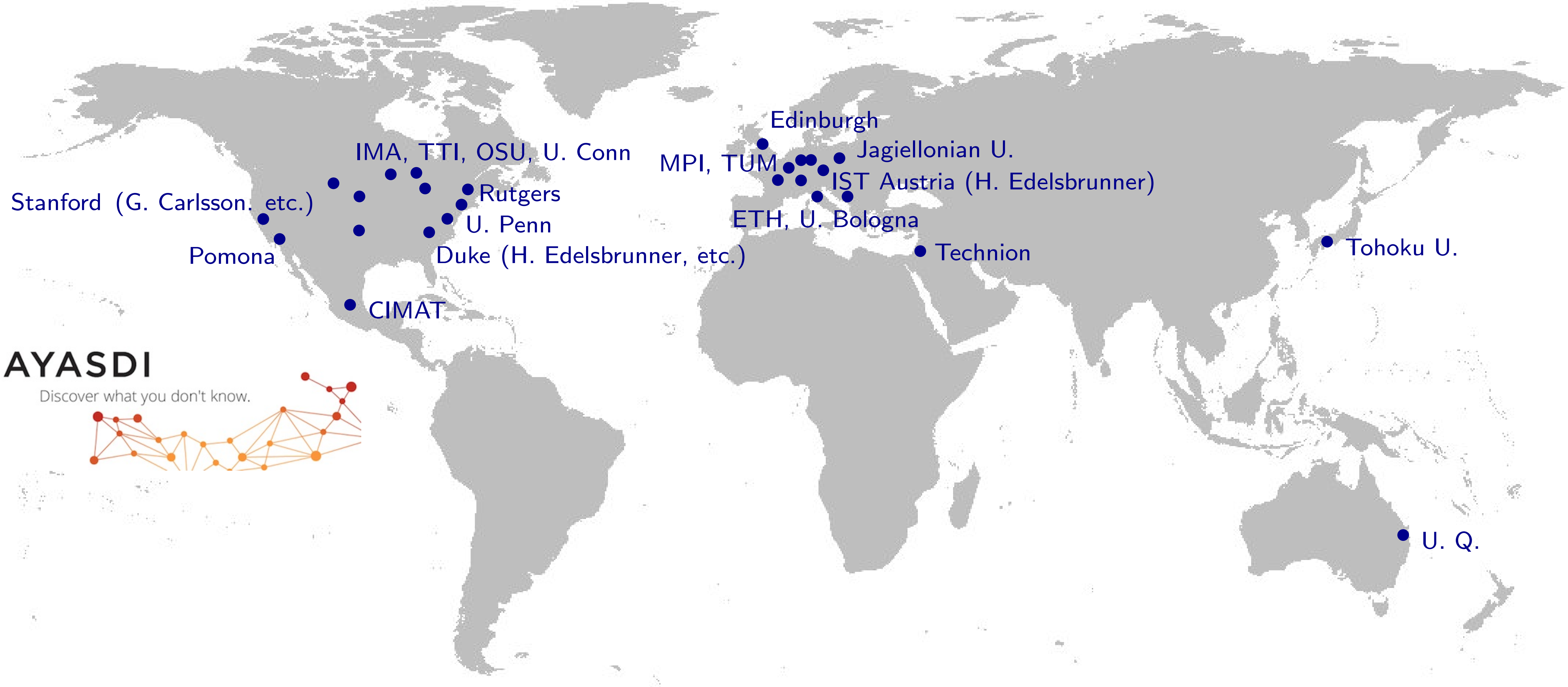


The TDA community (as of 2002)



- 2 research groups (5-10 researchers)

The TDA community (as of 2016)



- 50-100 researchers working on theoretical foundations
- 200-300 researchers at the interface with applications
- very successful applications and company (Ayasdi)

Some applications

- analysis of random, modular and non-modular scale-free networks and networks with exponential connectivity distribution,
- analysis of social and spatial networks like neurons, genes, online messages, air passengers, Twitter, face-to-face contact, etc.,
- coverage and hole detection in wireless sensor fields,
- multiple hypothesis tracking on urban vehicular data,
- analysis of the statistics of high-contrast image patches,
- image segmentation,
- 1d signal denoising,
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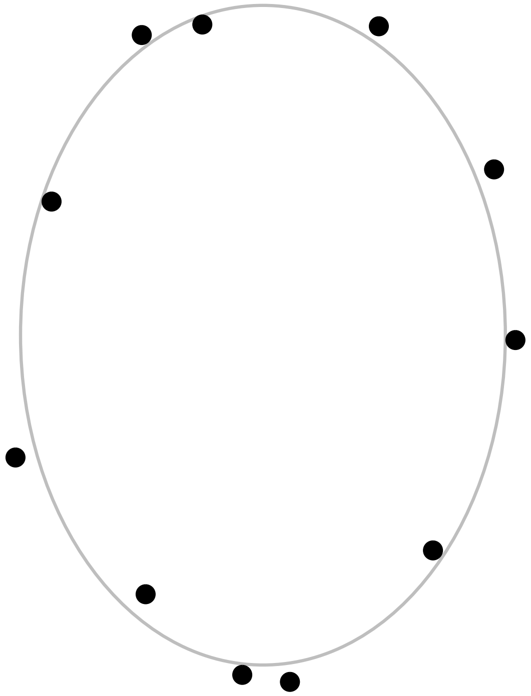
- refinement of the classification of NBA players,
- discrimination of electroencephalogram signals recorded before and during epileptic seizures,
- statistical analysis of orthodontic data,
- measurement of structural changes during lipid vesicle fusion,
- characterization of the frequency and scale of lateral gene transfer in pathogenic bacteria,
- pattern detection in gene expression data,
- study of the cosmic web and its filamentary structure,

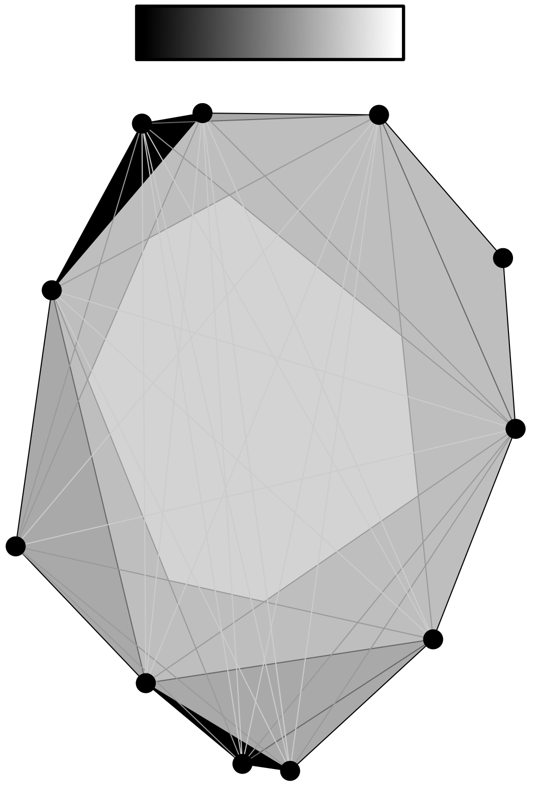
The TDA pipeline in a nutshell



point cloud P

proximity rule

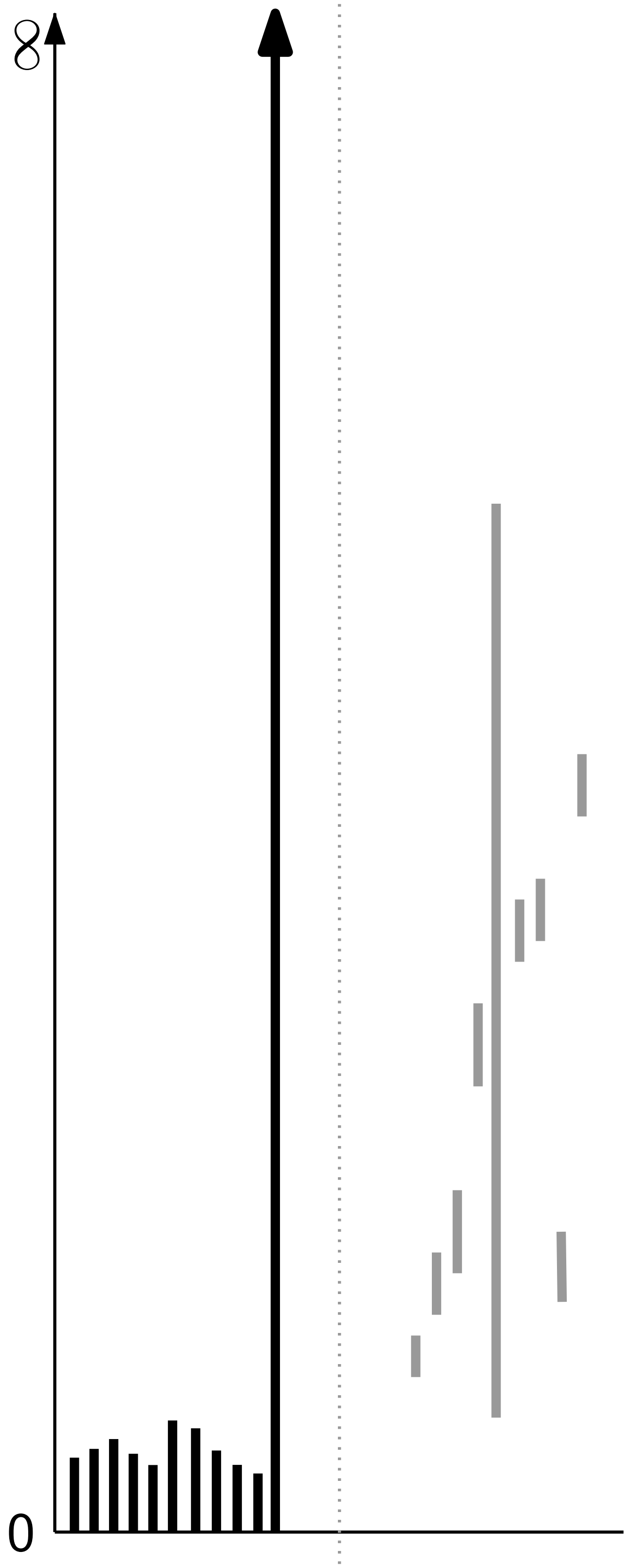
filter $P \to \mathbb{R}$



filtration $\mathcal{F}$

homology

persistence



barcode (descriptor)

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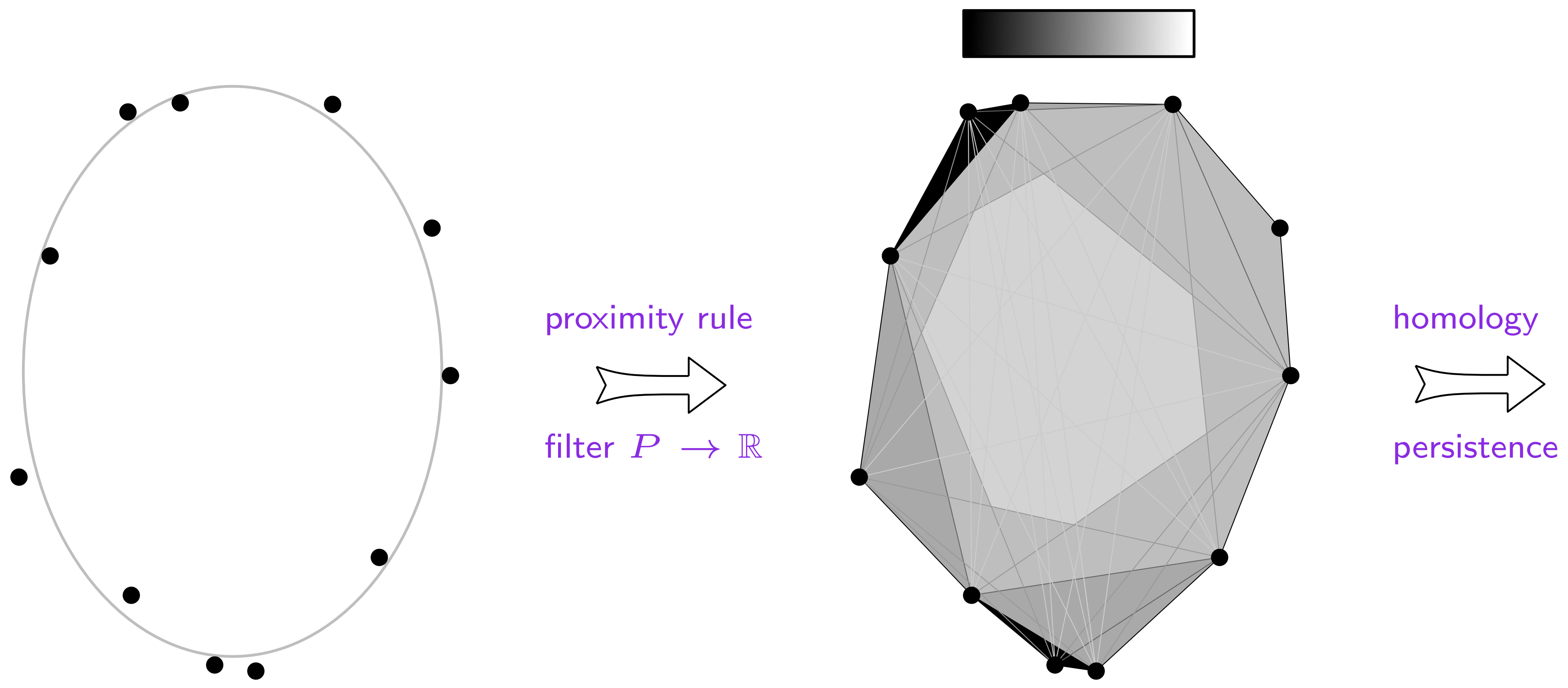

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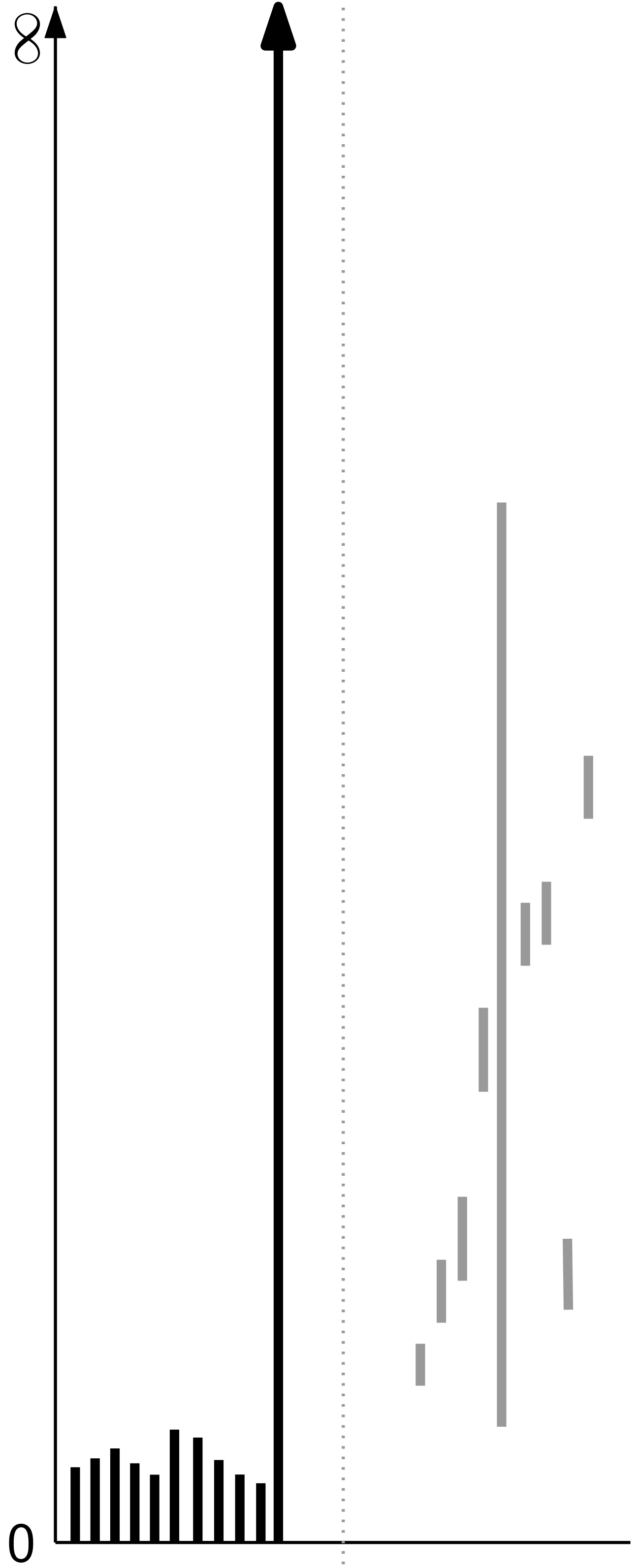

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3 pillars to the theory (topological persistence):

- decomposition theorems ($\exists$ barcodes)
- algorithms (computation of barcodes)
- stability theorems (barcodes as stable descriptors)

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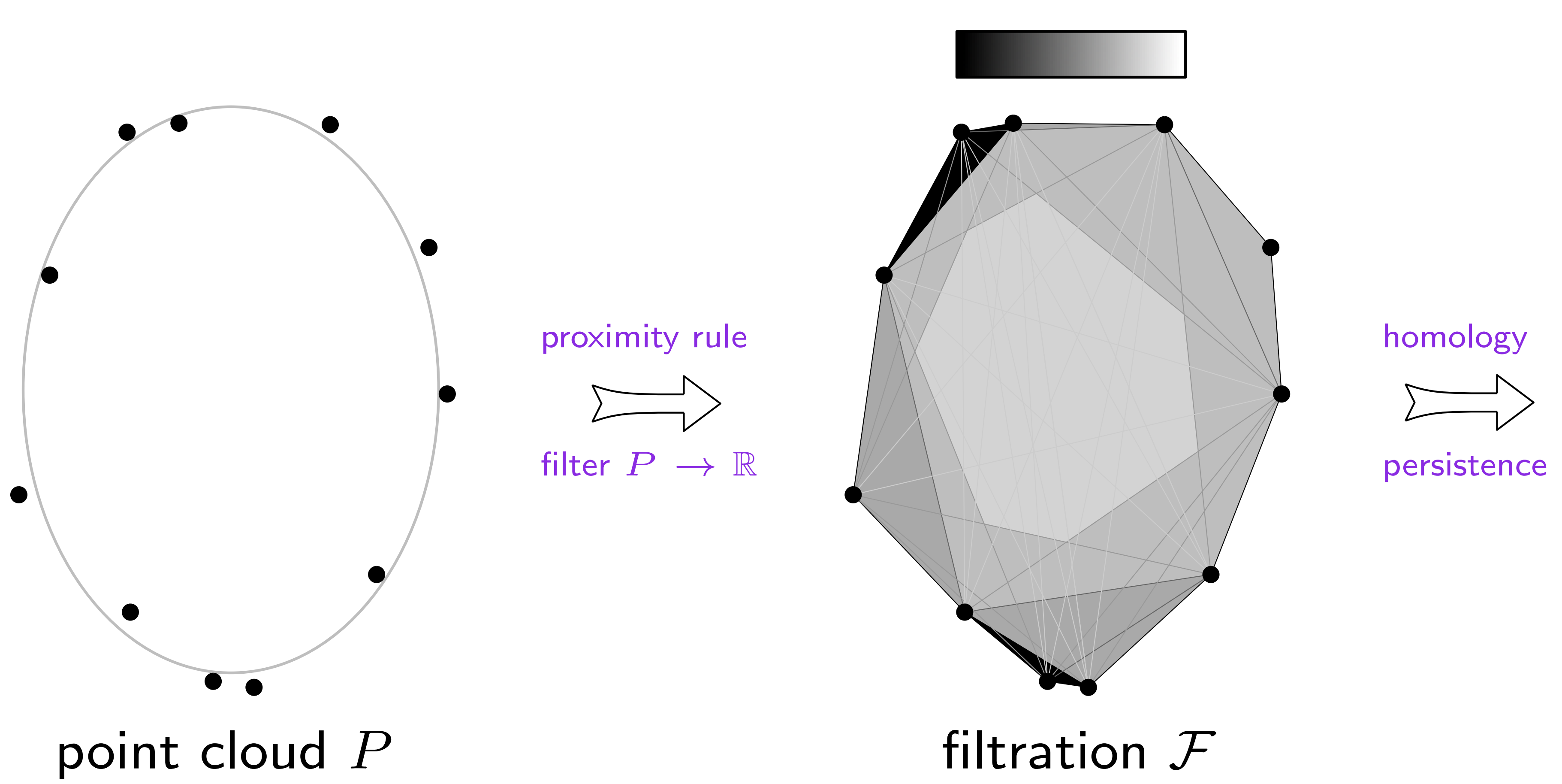

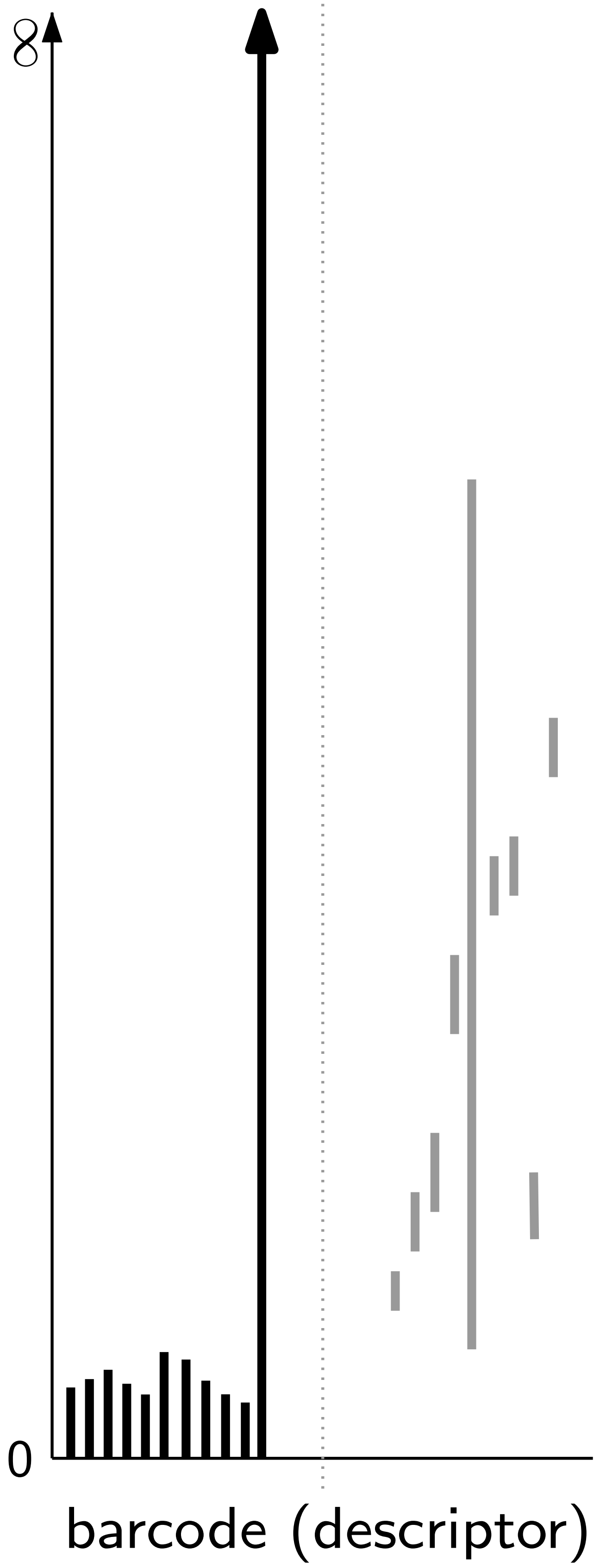

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