# Finite Element Analysis of Computer Aided Design Assembly

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# Outline

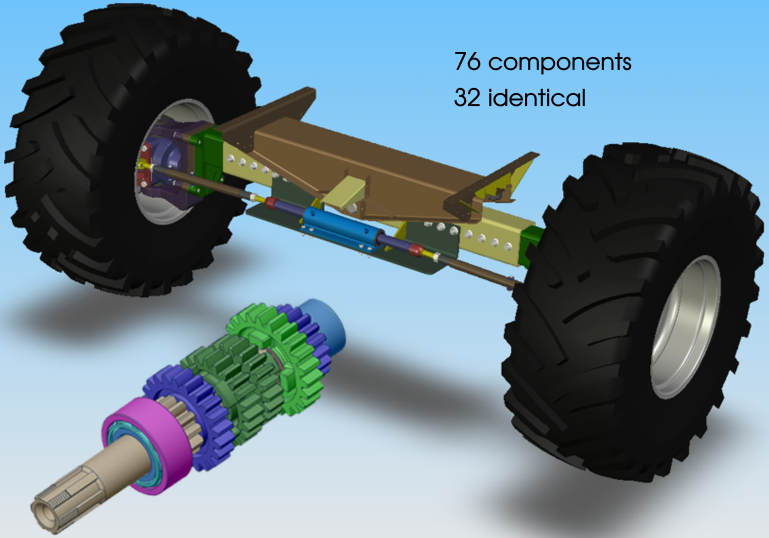
- CAD Modeling - Numerical Simulation :: Actual Situation
- CAD Based Domain Decomposition Method
    - Domain Decomposition
    - Nonoverlapping Methods
    - Geometric Discontinuity
    - Nonconforming Discretization
    - Solution Interpolation and Extension
- Numerical Illustrations
- Conclusions and Perspectives

http://www.gostaf.com

# CAD Assembly

76 components

32 identical



# Actual Situation

3D solid modeling

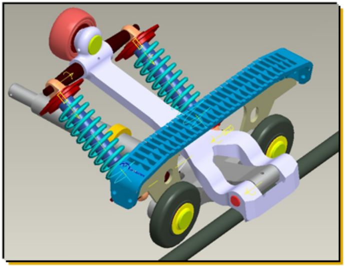
Kinematic and dynamic motion analysis

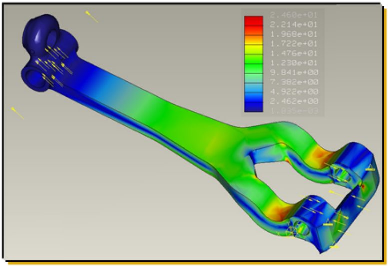
Capture time dependent displacements, reaction forces

Export entire assembly to single `step`, `iges` file

Loss of parametric advantages and feature based design

Artificial boundary conditions





# Actual Situation

- 3D solid modeling
- Kinematic and dynamic motion analysis
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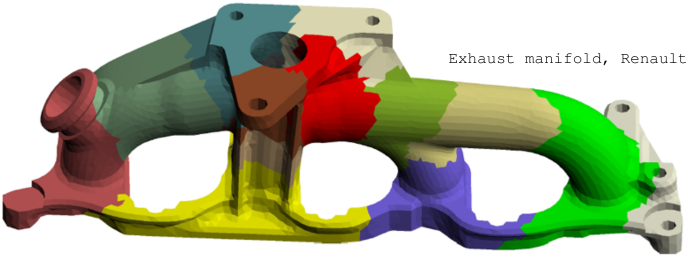
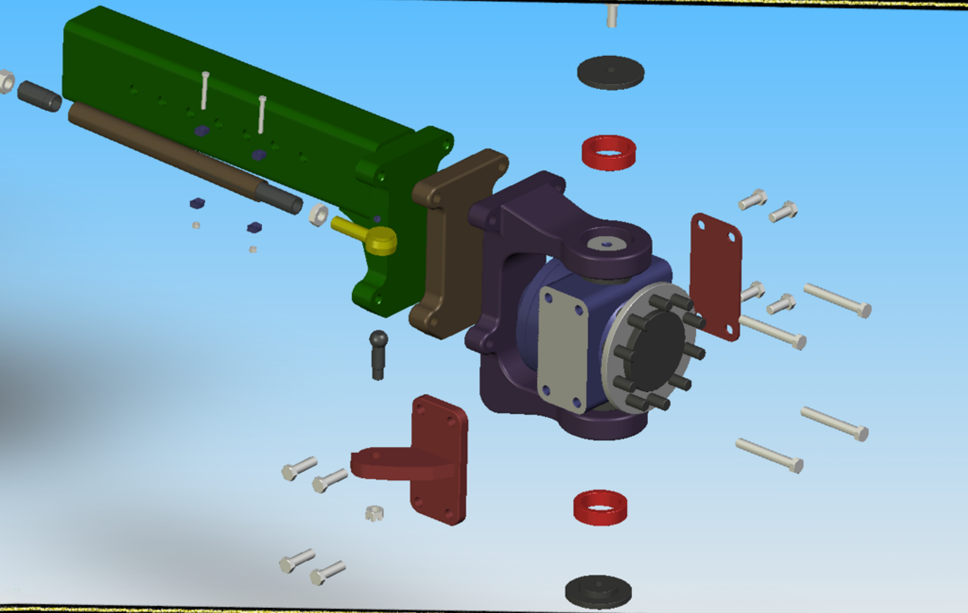
- Loss of parametric advantages and feature based design
- Artificial boundary conditions
- Domain decomposition - mathematical substructuring
- Parallel solution



Exhaust manifold, Renault

# CAD Assembly - Exploded View



# CAD Based Decomposition

Design oriented decomposition: materials, physical properties

Stable mathematical models, geometrical regularity of subdomains

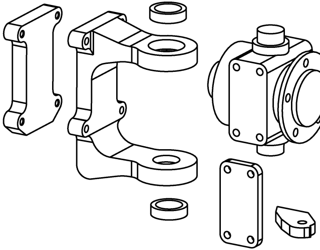
Contact regions generated by CAD, hight level of accuracy

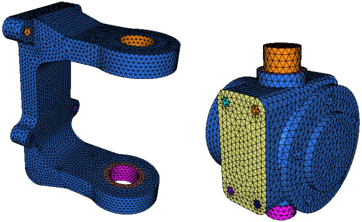
Mesh linked to the appropriate, independent geometry

Diverse element types and variational principles

Update of modified components, reuse of existing data

Parallel mesh generation

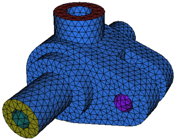




Model problem : Find $u \in V$ such that $\forall v \in V$

$$\int_\Omega E(x)\varepsilon(u) : \varepsilon(v) = \int_\Omega fv + \int_{\partial\Omega} fv$$

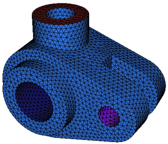
$E(x)$ elasticity tensor, $\varepsilon(u) = \frac{1}{2}(\nabla u + \nabla u^T)$



Suppose $\Omega$ is divided into $K$ subdomains :

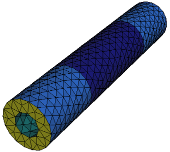
$$\overline{\Omega} = \cup_{k=1}^{K} \overline{\Omega}_k, \;\; \Omega_k \cap \Omega_l = \emptyset, \;\; k \neq l$$
$$S = \cup_{k=1}^{K} \partial\Omega_k \backslash \partial\Omega$$

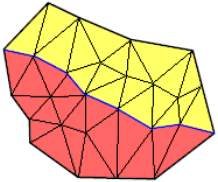
$u_k \in V_k$ - finite-dimensional space defined on $\Omega_k$



Hypothesis : If $u$ is known on $S$, the global problem could be reduced to $K$ local, independent problems

Recall : Geometrical continuity on skeleton





$u_1, u_2, u_3$ - internal nodes, interface

$$\begin{bmatrix} K_1 & 0 & K_{13} \\ 0 & K_2 & K_{23} \\ K_{13}^T & K_{23}^T & K_3 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix}$$

By substructuring, Schur complement matrix:

$$S\, u_3 = F$$

$$S \equiv K_3 - \sum K_{i3}^T\, K_i^{-1}\, K_{i3}$$
$$F \equiv f_3 - \sum K_{i3}^T\, K_i^{-1}\, f_i$$

Local solution :

$$u_i = K_i^{-1}(f_i - K_{i3}\, u_3)$$

Expensive for large scale problems or multiple subdomains

Consider an initial guess $u_s^0$ on the interface $S$ :

$$
\begin{aligned}
\mathcal{L}\, u_1^{n+1} &= f_1 \quad \text{in } \Omega_1 \\
u_1^{n+1} &= g_1 \quad \text{on } \partial\Omega_1 \backslash S \\
u_1^{n+1} &= u_s^n \quad \text{on } S
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}\, u_2^{n+1} &= f_2 \quad \text{in } \Omega_2 \\
u_2^{n+1} &= g_2 \quad \text{on } \partial\Omega_2 \backslash S \\
\frac{\partial u_2^{n+1}}{\partial n_2} &= -\frac{\partial u_1^{n+1}}{\partial n_1} \quad \text{on } S
\end{aligned}
$$

Correct the solution $u_s$ until conversionce, relaxation parameter $\theta$ :

$$u_s^{n+1} = (1-\theta)\, u_s^n + \theta\, u_2^{n+1}$$

Initial solution $u_s^0$ on $S$ :

$$\begin{aligned}
\mathcal{L}\, u_k^{n+1} &= f_k && \text{in } \Omega_k \\
u_k^{n+1} &= g_k && \text{on } \partial\Omega_k \backslash S \\
u_k^{n+1} &= u_s^n && \text{on } S
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}\, \psi_k^{n+1} &= 0 && \text{in } \Omega_k \\
\psi_k^{n+1} &= 0 && \text{on } \partial\Omega_k \backslash S \\
\partial_n \psi_k^{n+1} &= [\partial_n u_{k,l}^{n+1}] && \text{on } S
\end{aligned}$$

$$u_s^{n+1} = u_s^n - \theta\,(\psi_k^{n+1} + \psi_l^{n+1})$$

# Dual Method :: Dirichlet-Dirichlet

Initiale solution $u_s^0$ on $S$ :

$$
\begin{aligned}
\mathcal{L}\, u_k^{n+1} &= f_k && \text{in } \Omega_k \\
u_k^{n+1} &= g_k && \text{on } \partial\Omega_k \backslash S \\
u_k^{n+1} &= u_s^n && \text{on } S
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}\, \psi_k^{n+1} &= 0 && \text{in } \Omega_k \\
\psi_k^{n+1} &= 0 && \text{on } \partial\Omega_k \backslash S \\
\partial_n \psi_k^{n+1} &= [\partial_n u_{k,l}^{n+1}] && \text{on } S
\end{aligned}
$$

$$
u_s^{n+1} = u_s^n - \theta\,(\psi_k^{n+1} + \psi_l^{n+1})
$$

Initial flux $\lambda^0$ on $S$ :

$$
\begin{aligned}
\mathcal{L}\, u_k^{n+1} &= f_k && \text{in } \Omega_k \\
u_k^{n+1} &= g_k && \text{on } \partial\Omega_k \backslash S \\
\partial_n u_k^{n+1} &= \lambda^n && \text{on } S
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}\, \psi_k^{n+1} &= 0 && \text{in } \Omega_k \\
\psi_k^{n+1} &= 0 && \text{on } \partial\Omega_k \backslash S \\
\psi_k^{n+1} &= [u_{k,l}^{n+1}] && \text{on } S
\end{aligned}
$$

$$
\lambda^{n+1} = \lambda^n - \theta(\partial_n \psi_k^{n+1} - \partial_n \psi_l^{n+1})
$$

Find $u$ which minimizes : $J(v) = a(v,v) - (f,v) - (\lambda, v)_\Gamma$

$$\begin{bmatrix} K_1 & 0 & B_1^T \\ 0 & K_2 & -B_2^T \\ B_1 & -B_2 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ \lambda \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ 0 \end{bmatrix}$$

$B_i$ - connectivity boolean matrix, $\lambda$ - Lagrange multiplier

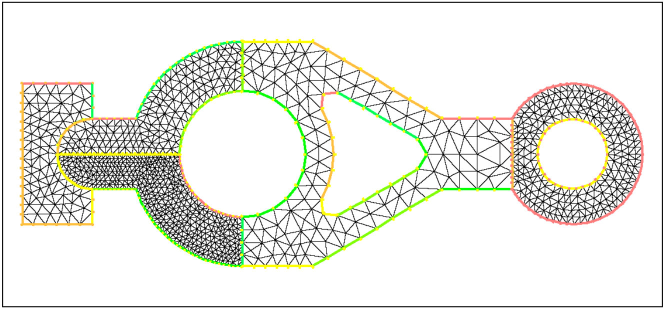
The Mortar operator :

$$\Phi = Tr(u_{|\Omega_k^-}) \quad 1 \le k \le K$$

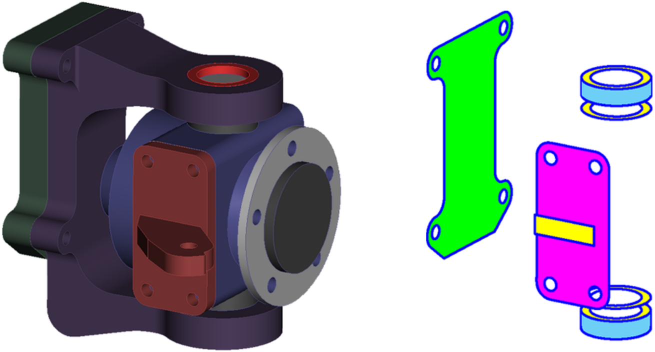
Matching condition on each non-mortar (weak sense) :

$$\forall \psi \in W, \quad \int_{\gamma^+} \left(Tr(u_{|\Omega_k^+}) - \Phi(u)\right) \psi \, d\gamma = 0$$

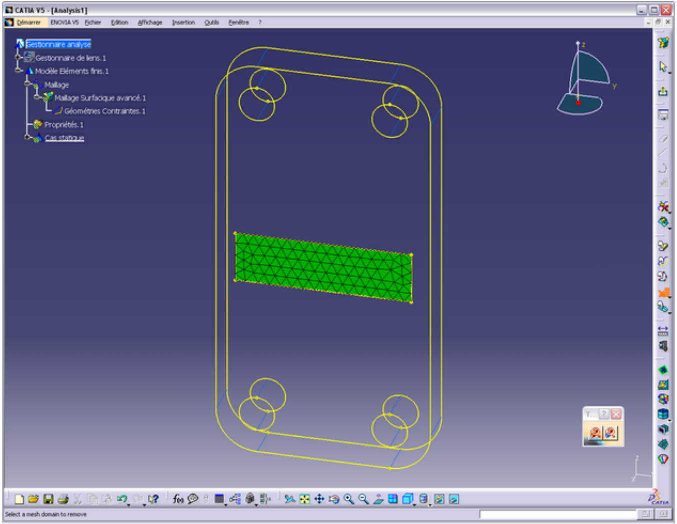
5 domains :: Linear Elasticity



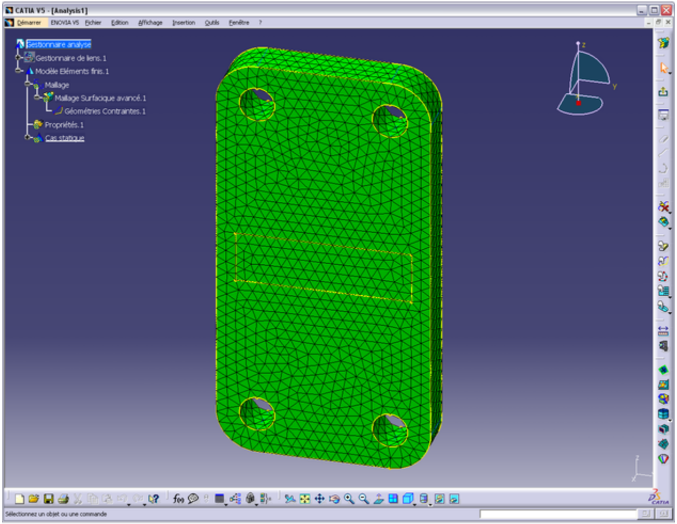
# CAD Driven Contact Surfaces



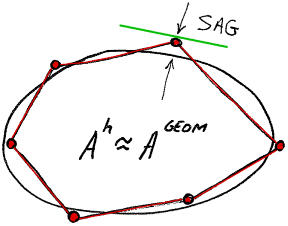
# Conforming Mesh

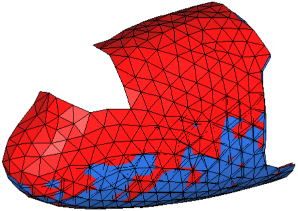


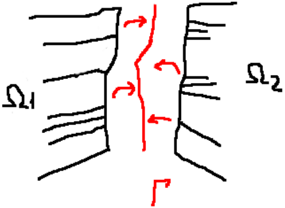
# Conforming Mesh



# Nonconforming Discretization






Border integral : $\int_\Gamma e_i^{\Omega_k} \, e_j^{\Omega_l} \, d\Gamma$

Fast non-conforming interface projections, *M.Gander*

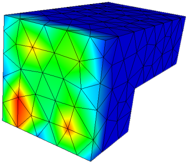
A mortar segment-to-segment frictional contact method for large deformations, *T.Laursen et.al.*

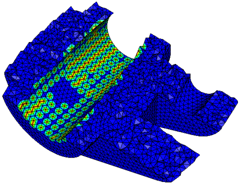
# Geometric Discontinuity

Freefem++ 3D version

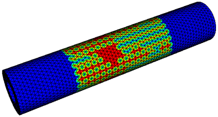
```
fespace Vh1(D1,P13d); fespace Vh2(D2,P13d);
fespace Vhs(D1,P13d);
Vh1 u1,v1; Vh2 u2,v2; Vhs Lam;

problem pb1(u1,v1) = pbDefine(1) + int2d(D1,1)(Lam*v1);

problem pb2(u2,v2) = pbDefine(2) + int2d(D2,1)(-Lam*v2);
```

Barycentric coordinates : $-\epsilon < \lambda_i < 1 + \epsilon$

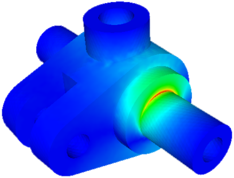




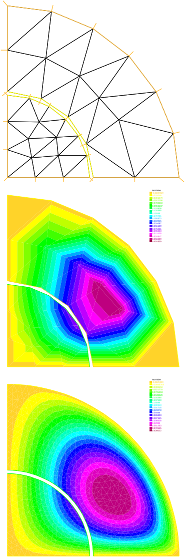
FreeFem3D



FreeFem++



# Solution Extension

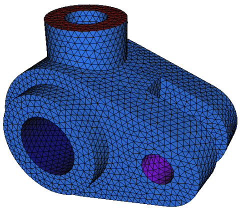


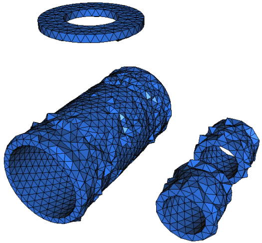
Extension $I_k : V_k \mapsto V$

- by zero
- linear
- virtual midpoint
- best neighbors

Augmented Skeleton -

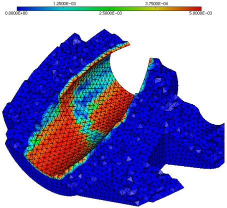
- set of elements that share border vertices

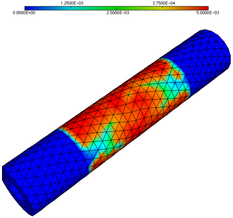




# Numerical Test :: Holder

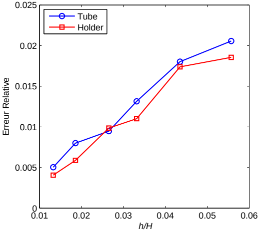
We study the influence of geometric discontinuity for curved contact boundaries. Gaps and intersections are of characteristic size $h$.

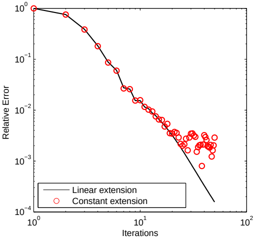


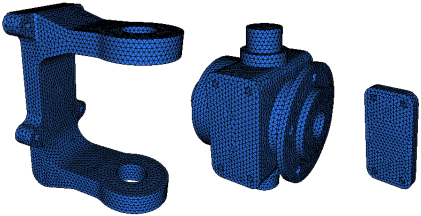

We verify the jump of the mono-domain solution across the boundary:

$$e = \left[\int_\gamma \left(u_{|\Omega_k} - \mathcal{M}_{l\to k} u_{|\Omega_l}\right)^2 d\gamma\right]^{\frac{1}{2}}$$



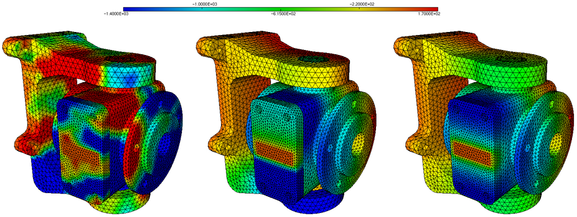
# Numerical Test :: Chassis

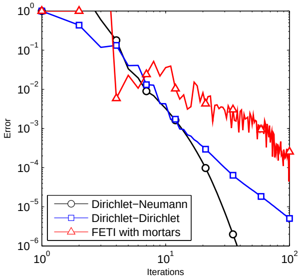


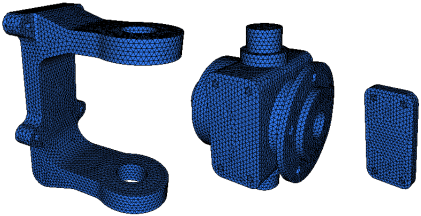
Problem of linear elasticity is solved.
Displacements imposed on partition of boundary for each subdomain.
Linear solution extension.

Node 2.4GHz, 1Gb. CPU 32sec/iter.

$D_1$:51814, $D_2$:108790, $D_3$:23192 elements.





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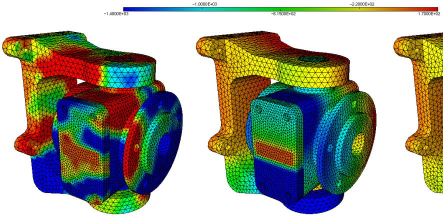


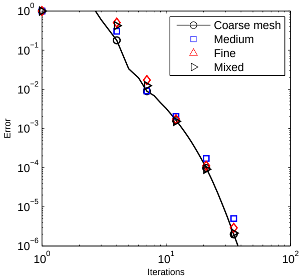

# Conclusions and Perspectives

CAD based automatic substructuring, contact surfaces

Component dependent FE model /element types, variational principals

Mesh linked to geometry

Diverse types of solution extension

Error convergence independent of mesh refinement

Parallelism and modularity

Numerical tests for large number of subdomains

Contact treatment: sliding, collision