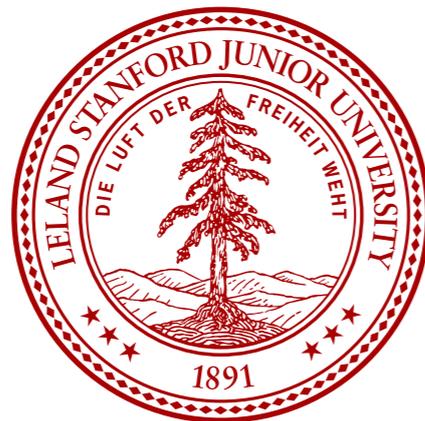


<http://www-sop.inria.fr/geometrica/collaborations/TGDA/index.html>

# TGDA

## Topological and Geometric Data Analysis

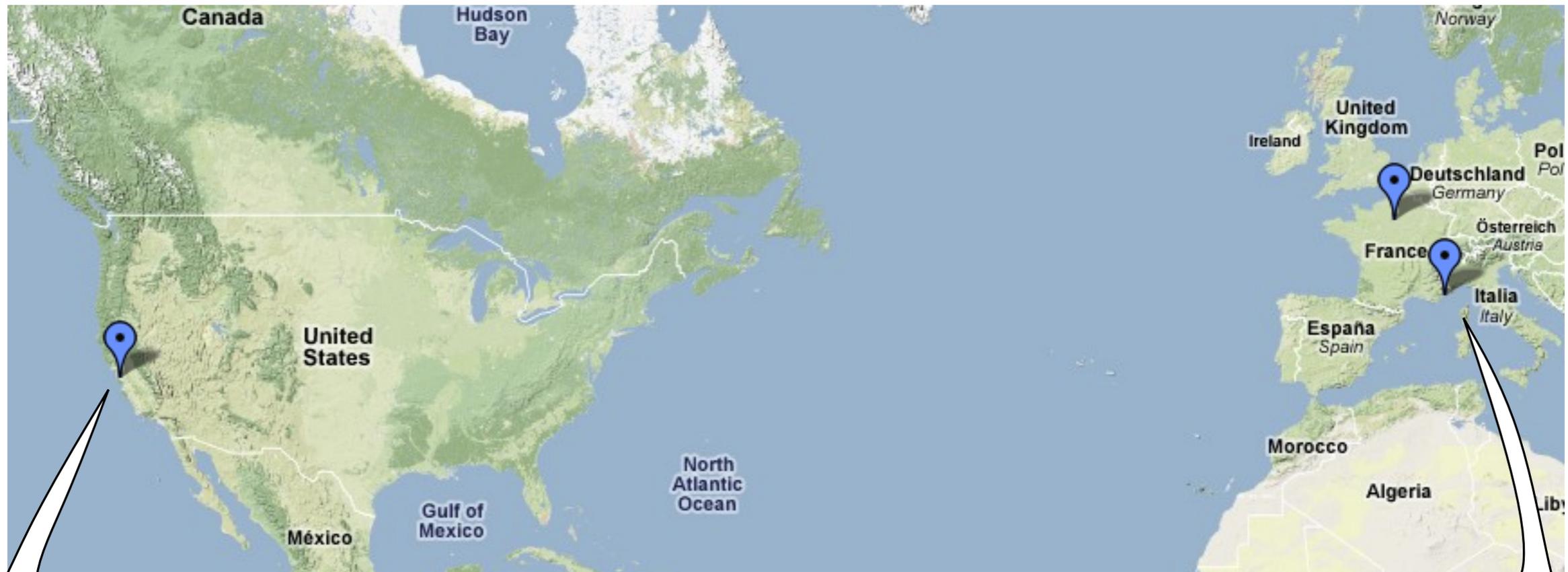
Geometric Computing group  
Stanford University



Geometrica group  
INRIA Saclay / Sophia



# Official collaboration between research groups



## - Geometric Computing group

(L. J. Guibas, 15 people)

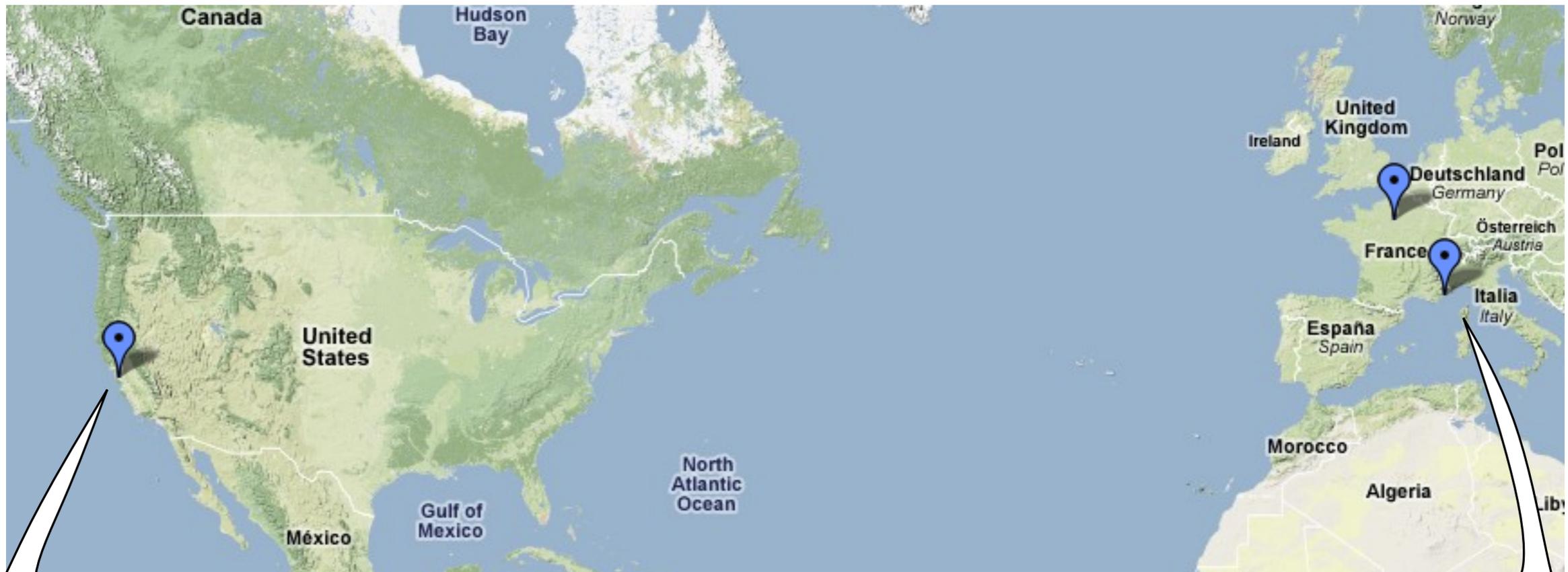


## - Geometrica group

(J-D Boissonnat, 25 people)



# Official collaboration between research groups



## - **Geometric Computing group**

(L. J. Guibas, 15 people)

- Applied Topology group

- ML, Robotics, Bio-computation...



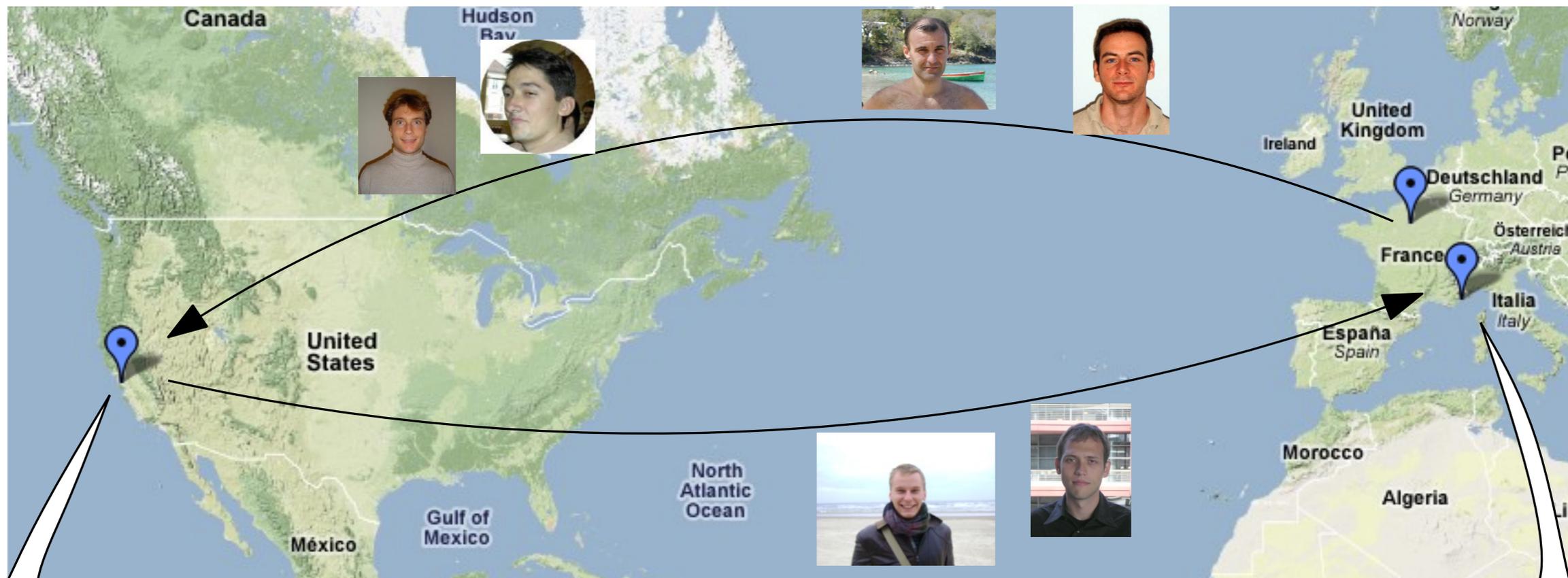
## - **Geometrica group**

(J-D Boissonnat, 25 people)

- ABS, Select, Asap...



# Exchanges (2008)



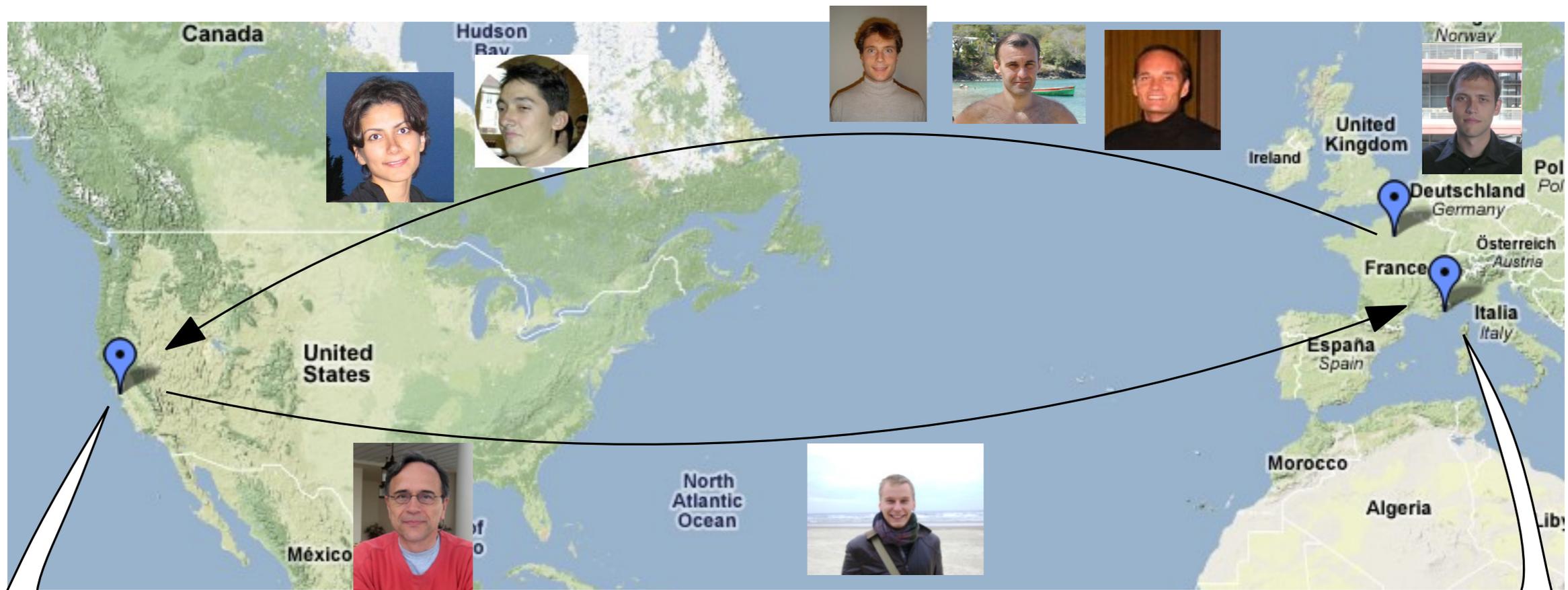
## - Geometric Computing group

- long-term: M. Ovsjanikovs, P. Skraba

## - Geometrica group

- long-term: Q. Mérigot, S. Oudot
- short-term: F. Chazal, D. Cohen-Steiner

# Exchanges (2009)



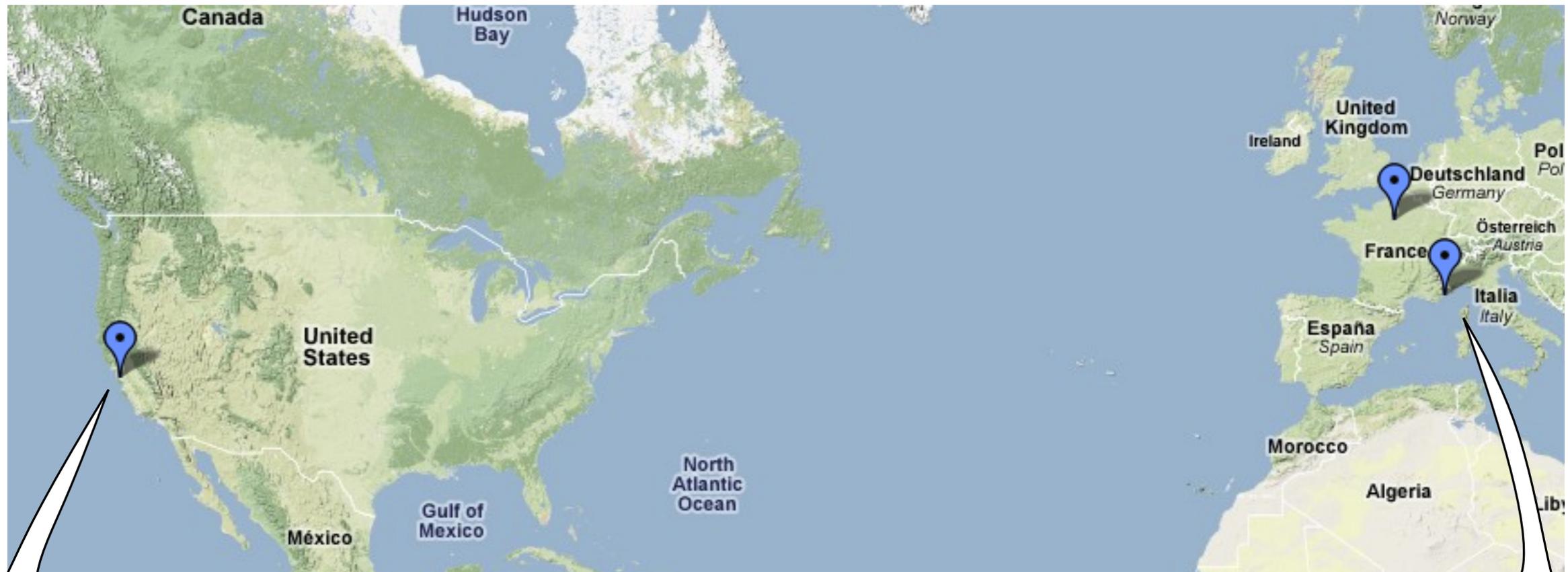
## - Geometric Computing group

- long-term: M. Ovsjanikovs
- short-term: L. Guibas
- 2-year post-doc: P. Skraba

## - Geometrica group

- long-term: Q. Mérigot, P. Memari
- short-term: F. Chazal,  
J-D Boissonnat,  
S. Oudot

# Funding



## - Geometric Computing group

18 kE in 2008 (NSF)

25 kE in 2009 (NSF + FFS)

■ ■ ■

## - Geometrica group

20 kE in 2008 (EA)

20 kE in 2009 (EA)

■ ■ ■

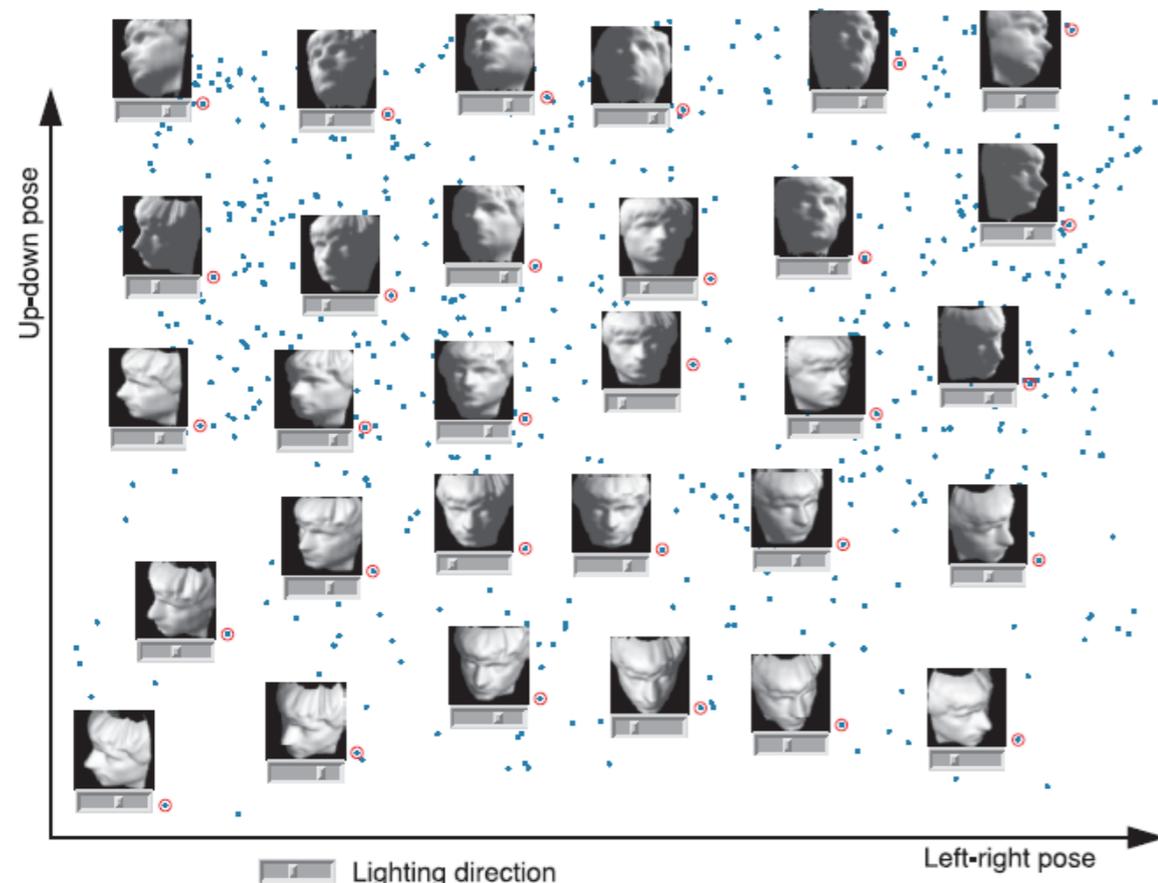
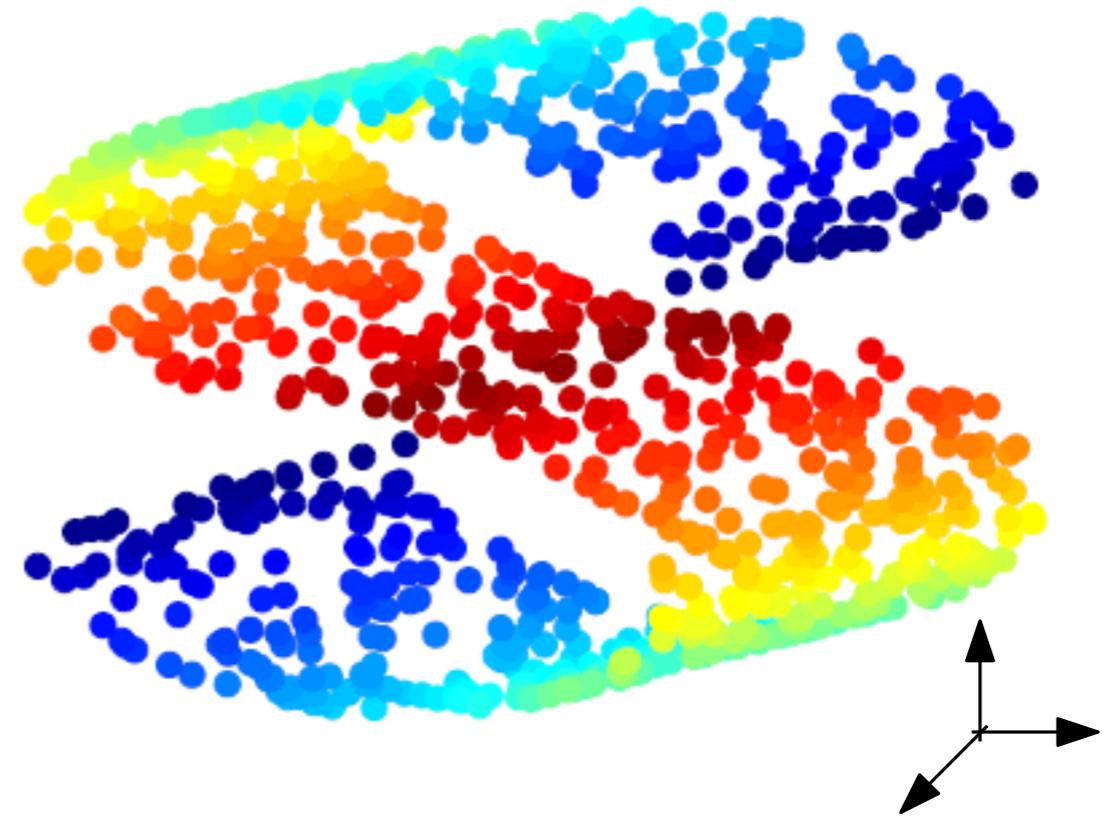
# Main Scientific Goals

**Input:** a point cloud in a metric space.

Is there structure in the data?

Can we infer topological invariants?

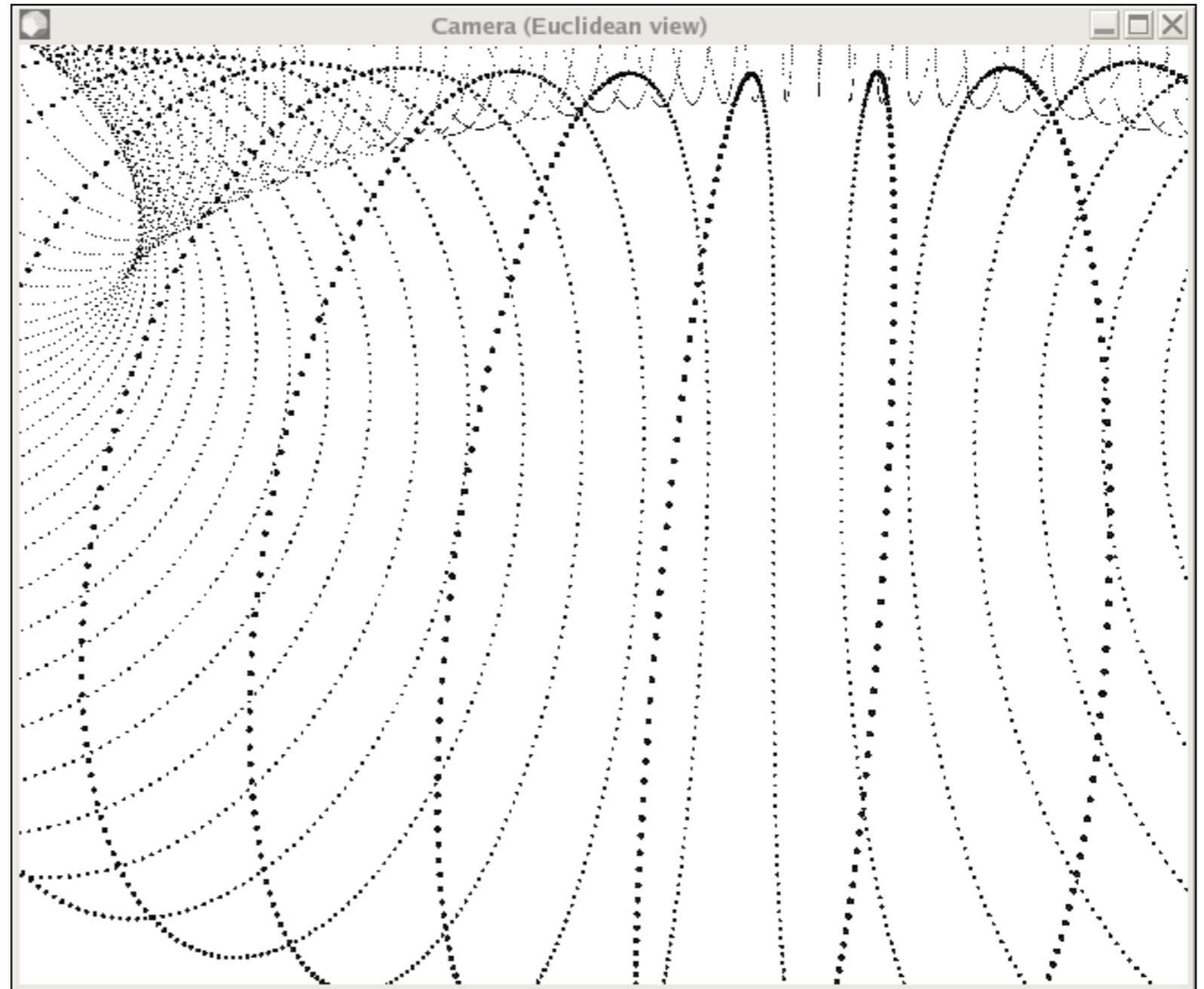
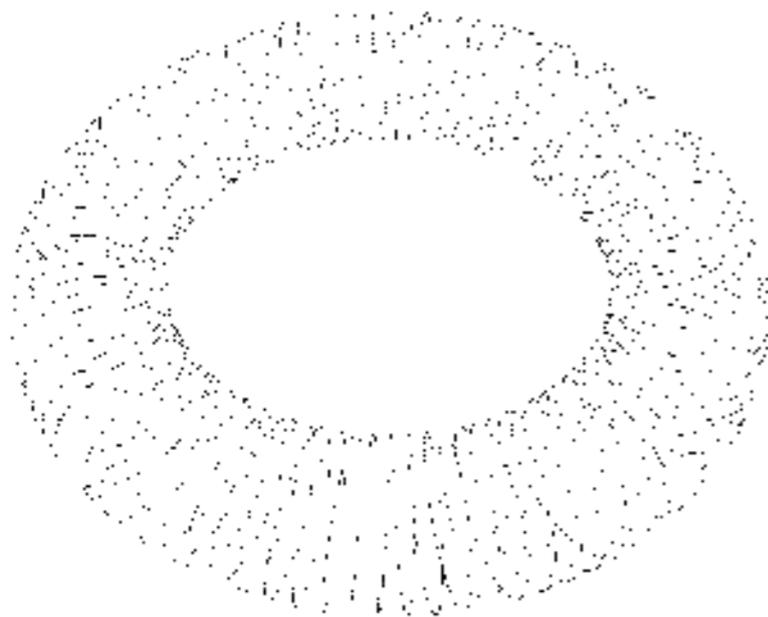
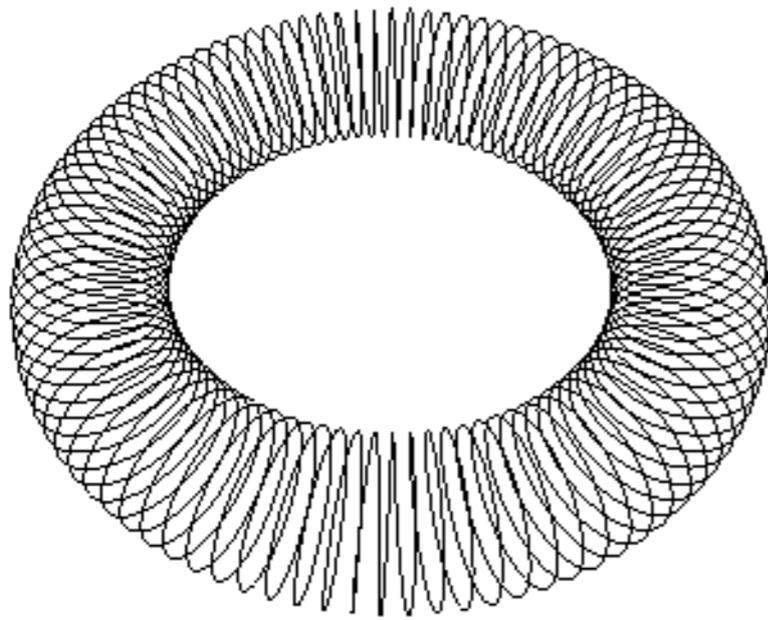
Can we approximate differential quantities?



Example: set of 4096-dimensional data points, representing 64x64 pixels images of a same object, seen under various lighting and camera angles. (from Isomap, *Science* 290).

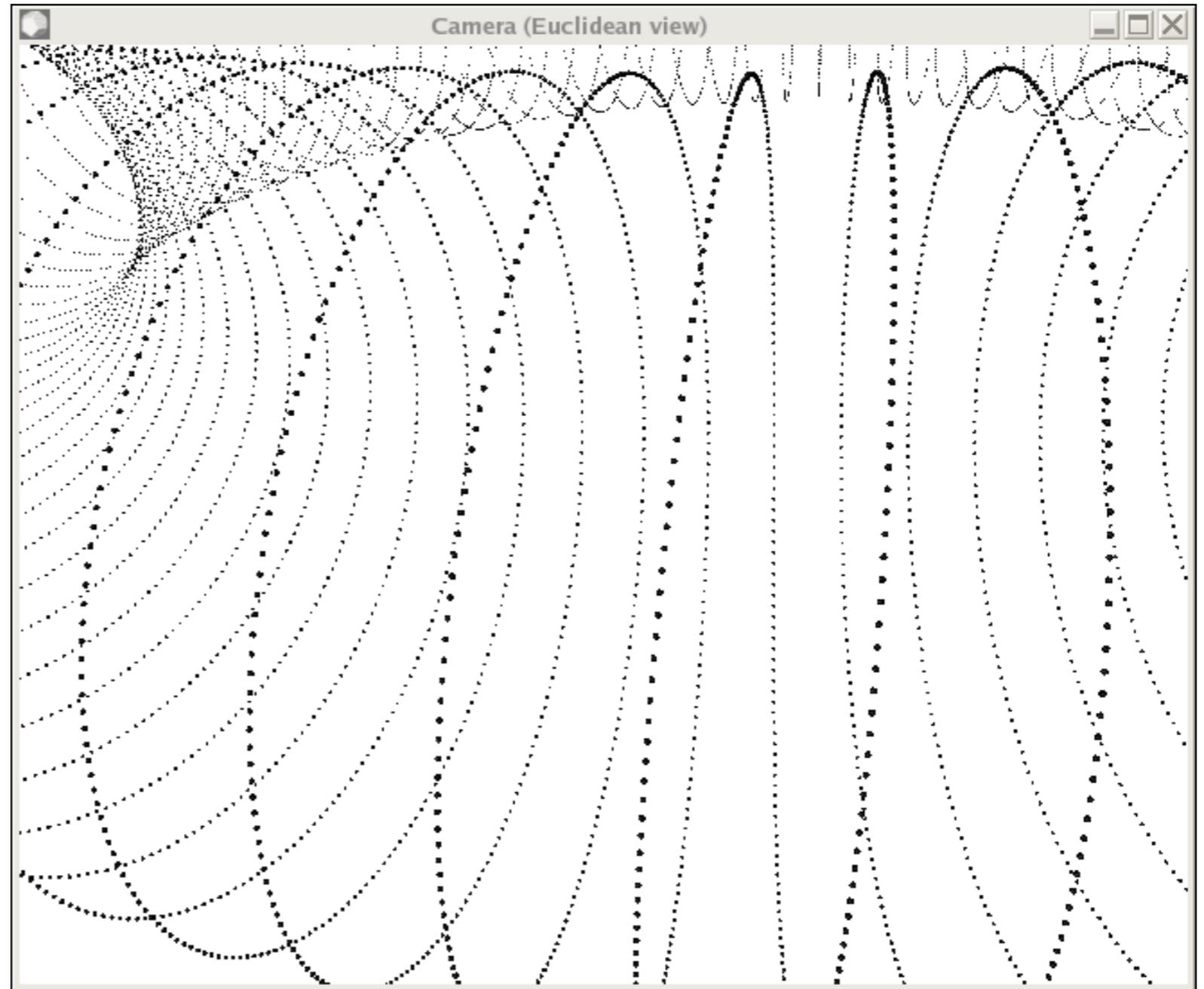
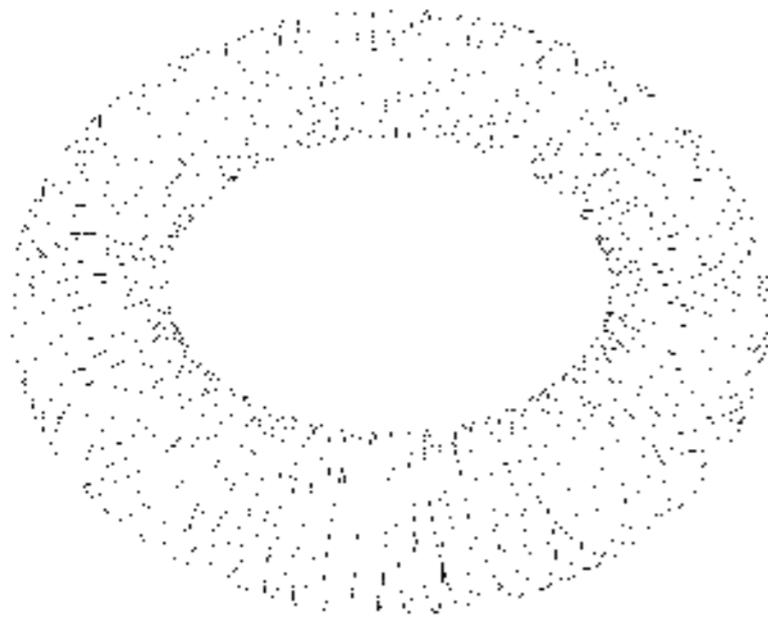
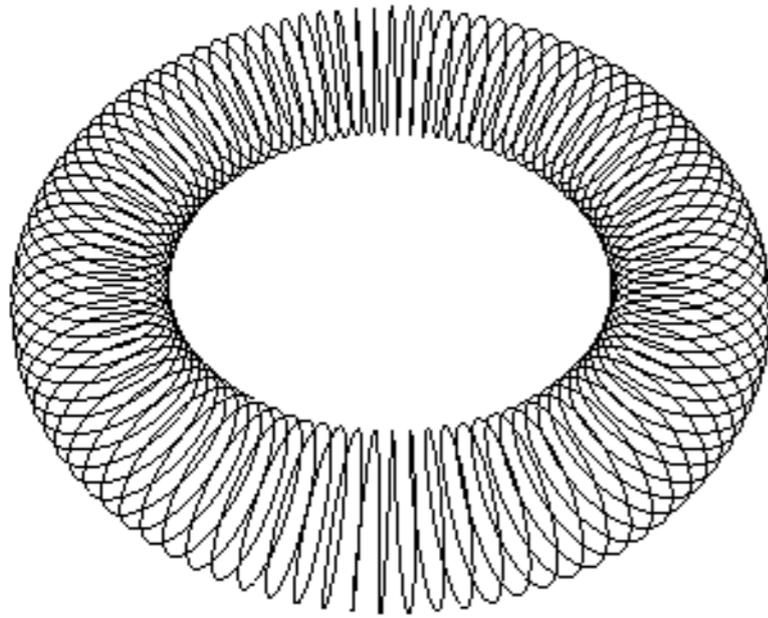
# Theoretical Challenges

What is the underlying object?



# Theoretical Challenges

What is the underlying object?



- perform multi-scale analysis (landmarking, topological persistence)

# Algorithmic Challenges

## Curse of dimensionality:

*The  $\varepsilon$ -entropy ( $\varepsilon < 1$ ) of a smooth  $k$ -dimensional submanifold  $M$  of Euclidean space  $\mathbb{R}^d$  is of the order of  $-k \log(\varepsilon)$ . In other words, for any finite set  $X$  s.t.  $d_H(M, X) \leq \varepsilon$ ,  $|X| = \Omega((\frac{1}{\varepsilon})^k)$ .*

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- assume high co-dimension:  $1 \lesssim k \ll d$

## Double curse:

*For a finite set of points on a  $k$ -submanifold of  $\mathbb{R}^d$ , classical data structures from computational geometry and topology (Čech complex, Delaunay triangulation,  $\alpha$ -shape) scale up exponentially with  $d$ , not  $k$ .*

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- Build lightweight data structures (Rips complex, witness complex)

# Theoretical Tools

## Delaunay

- restricted Delaunay
- $\varepsilon$ -sampling theory
- $\alpha$ -shape
- Witness complex

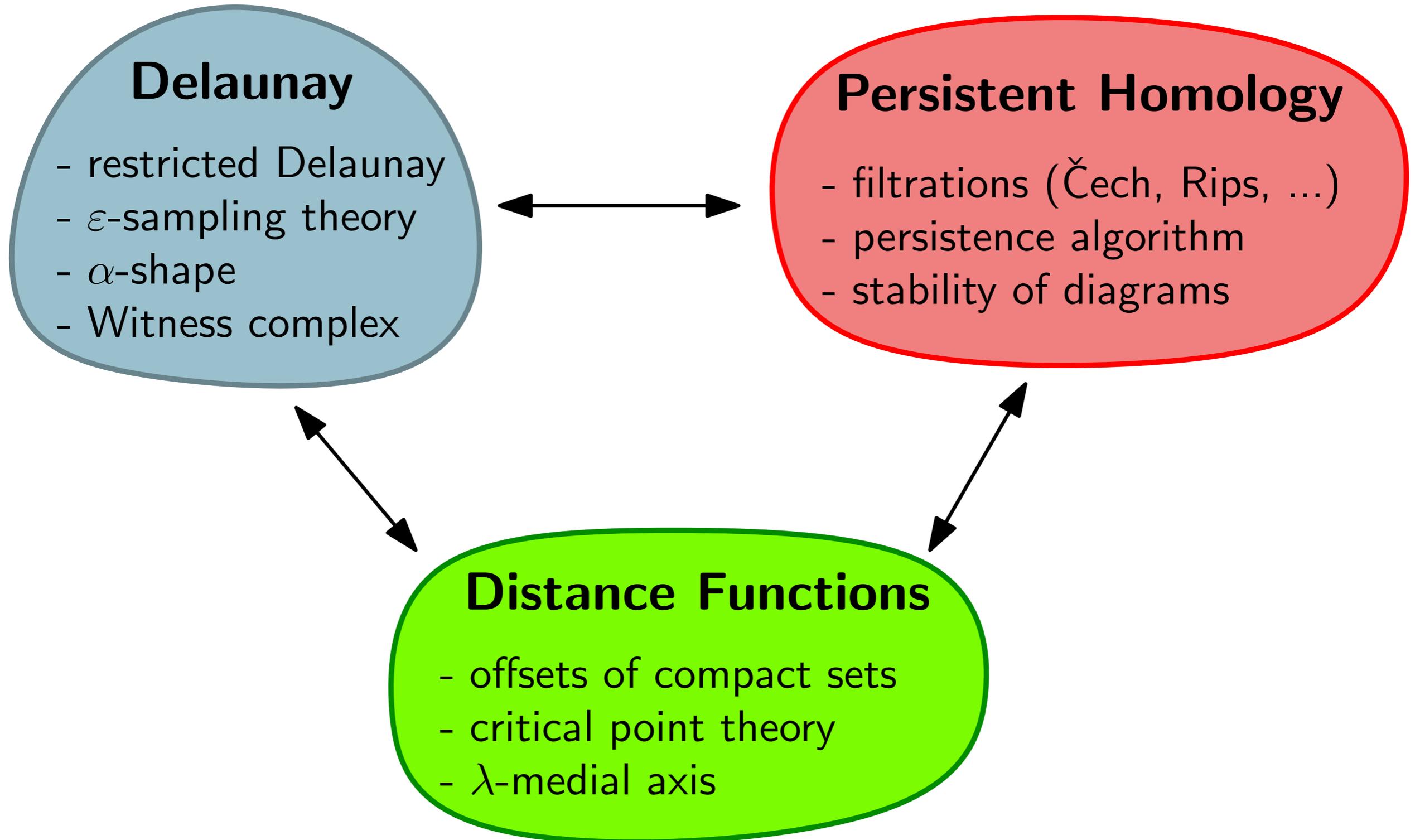
## Persistent Homology

- filtrations (Čech, Rips, ...)
- persistence algorithm
- stability of diagrams

## Distance Functions

- offsets of compact sets
- critical point theory
- $\lambda$ -medial axis

# Theoretical Tools



# Some Achievements and Perspectives

Wednesday, July 8th		Thursday, July 9th	
08:30 - 09:00	Registration	09:00 - 09:20	Registration
09:00 - 09:20	Opening remarks	09:20 - 10:30	<b>Key Note:</b> An introduction to zigzag persistence Vin de Silva
09:20 - 10:30	<b>Key Note:</b> Geometric entropy minimization Alfred Hero	10:30 - 10:50	Coffee break
10:30 - 10:50	Coffee break	10:50 - 12:30	<b>Session 4: Persistence and unsupervised learning</b>
10:50 - 12:30	<b>Session 1: Reconstruction in 3D</b>		Persistence-based clustering Primož Skraba
	Scale space meshing Julie Digne		Persistent cohomology and circular coordinates Mikael Vejdemo-Johansson
	Reconstructing 3D compact sets Frédéric Cazals	12:30 - 14:00	Lunch
12:30 - 14:00	Lunch	14:00 - 15:40	<b>Session 5: Signatures for shape classification</b>
14:00 - 15:40	<b>Session 2: Reconstruction in arbitrary dimensions</b>		Topo-geometric Modeling for 3D objects Hamid Krim
	Manifold Reconstruction from Tangential Complex Arijit Ghosh		Gromov-Wasserstein stable signatures for object matching and the role of persistence Facundo Mémoli
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